

THE
PRINCIPLES OF HYDROSTATICS:

DESIGNED
FOR THE USE OF STUDENTS
IN THE
UNIVERSITY.

BY THE REV. S. VINCE, A.M. F.R.S.
PLUMIAN PROFESSOR OF ASTRONOMY AND EXPERIMENTAL
PHILOSOPHY.

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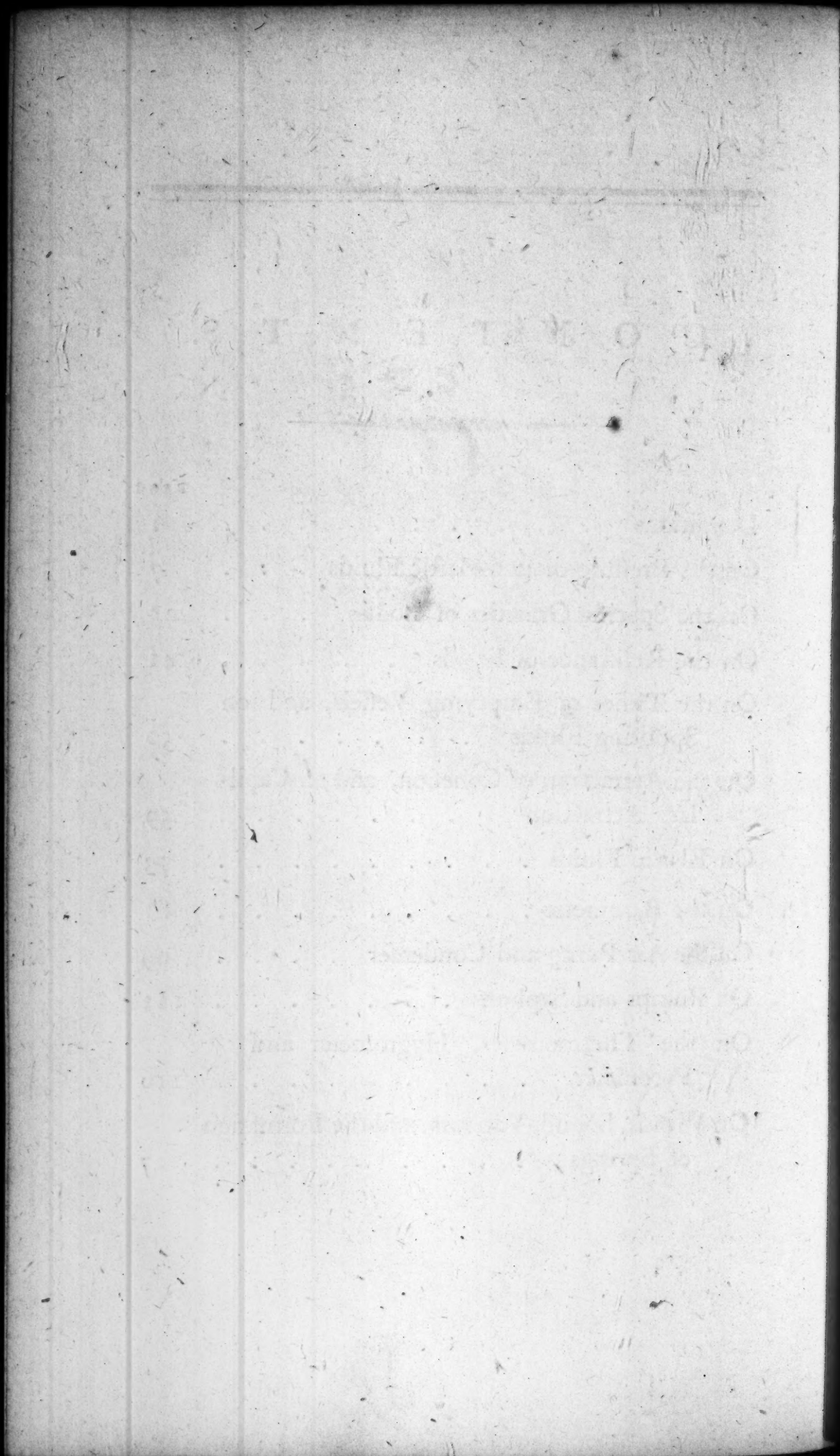
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C O N T E N T S.

	PAGE
Definitions	I
On the Pressure of non-elastic Fluids	7
On the Specific Gravities of Bodies	25
On the Resistance of Fluids	41
On the Times of Emptying Vessels, and on Spouting Fluids	50
On the Attraction of Cohesion, and on Capil- lary Attraction	59
On Elastic Fluids	73
On the Barometer	81
On the Air Pump and Condenser	99
On Pumps and Syphons	111
On the Thermometer, Hygrometer and Pyrometer	116
On Winds, Sound, Vapours, and the Formation of Springs	127



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THE

PRINCIPLES OF HYDROSTATICS.

DEFINITIONS.

Art. 1. **T**HE science which treats of the nature and properties of Fluids has been usually divided into the following branches; *Hydrostatics*, which comprises the doctrine of the equilibrium of *non-elastic* fluids, as water, mercury, &c.; *Hydraulics*, which relates to the motion of those fluids; and *Pneumatics*, which treats on the properties of the different kinds of airs. But these are now all included under the general term *Hydrostatics*.

2. A *Fluid* is a body whose parts are put in motion one amongst another by any force impressed; and which, when the impressed force is removed, restores itself to it's former state *.

Fluids may be divided into *elastic* and *non-elastic*.

An

* By it's *former state*, we do not mean that every particle is re-instated in it's former situation, but that the whole body recovers it's former dimensions and figure.

An *elastic* fluid is one, whose dimensions are diminished by increasing the pressure, and increased by diminishing the pressure upon it; of which description are the different kinds of airs. A *non-elastic* fluid is one, whose dimensions are not, at least as to sense, affected by any increase of pressure, as water, mercury, &c. As many bodies, by cold, from a state of fluidity become solids, such bodies are fluids so long as their surfaces, when disturbed, will restore themselves to their former position. The definition supposes a *partial* pressure; for if the fluid be incompressible, under an equal and general pressure none of the parts will be moved. Different fluids have different degrees of fluidity, according to the facility with which the particles are moved one amongst another. Water and mercury are the most perfect non-elastic fluids. Many fluids have a very sensible degree of tenacity, and are therefore called imperfect fluids. Besides the fluids which come under this definition, there are others, as the electric and magnetic fluid, light, and fire, according to the opinion of some, &c. but these are not the objects of Hydrostatics.

3. The *specific gravity* of a body is it's weight, compared with the weight of another body whose magnitude is the same.

4. The *density* of a body is as the quantity of matter contained in a given space, and therefore (Mech. Art. 26) in proportion to it's weight, when the magnitude is the same.

5. Cor. Hence the *specific gravity* of a body is in proportion to it's *density*.

A cubic inch of pure mercury is about fourteen times heavier than a cubic inch of water; the specific gravity

gravity and density of the former are therefore about fourteen times that of the latter. As the weight of a body is in proportion to it's quantity of matter (Mechanics, Art. 26), the specific gravity and density of a body are also in proportion to it's quantity of matter, when the magnitude is the same.

6. If the magnitude of a body be increased, the density remaining the same, the quantity of matter, and consequently the weight, will be increased in the same proportion. Hence if the magnitude M , and density D both vary, the quantity of matter, and consequently the weight of the body, will vary as $M \times D$, by the composition of ratios.

7. We know so little of the nature and constitution of fluids, that the application of the general principles of motion to the investigation of the effects produced by their action, is subject to great uncertainty. That the different kinds of airs are constituted of particles endued with repulsive powers, is manifest from their expansion, when the force with which they are compressed is removed. The particles being kept at a distance by their mutual repulsion, it is easy to conceive that they may move very freely amongst each other, and that this motion may take place in all directions, each particle exerting it's repulsive power equally on all sides. Thus far we are acquainted with the constitution of these fluids; but with what degree of facility the particles move, and how this may be affected under different degrees of compression, are circumstances of which we are totally ignorant. With respect to the nature of those fluids which are denominated liquids, we are still less acquainted. If we suppose their particles to be in contact, it is very difficult

cult to conceive how they can move amongst each other with such extreme facility, and produce effects in directions opposite to the impressed force, without any sensible loss of motion. To account for this, the particles have, by some, been supposed to be perfectly smooth and spherical. If we were to admit this supposition, it would yet remain to be shown how it would solve all the phænomena, for it is by no means self-evident that it would. If the particles be not in contact, they must be kept at a distance by some repulsive power. But it is manifest that these particles attract each other, from the drops of all perfect fluids endeavouring to form themselves into spheres. We must therefore admit in this case both powers, and that where one power ends the other begins, agreeable to Sir I. NEWTON's * idea of what takes place, not only in respect to the constituent particles of bodies, but to the bodies themselves. The incompressibility of liquids (for I know no decisive experiments which have proved them to be compressible) seems most to favour the former supposition, unless we admit, in the latter hypothesis, that the repulsive force is greater than any human power which can be applied. The expansion of water by heat, and the possibility of actually converting it into two permanent elastic fluids, according to some late experiments, seem to prove that a repulsive power exists between the particles, for it is hard to conceive that heat can actually create any such new powers, or that it can of itself produce any such effects.

A fluid being composed of an indefinite number of corpuscles, we must consider it's action, either as the
joint

* See his Optics, Que. 31.

joint action of all the corpuscles, estimated as so many distinct bodies, or we must consider the action of the whole as a mass, or as one body. In the former case, the motion of the particles being subject to no regularity, or at least to none that can be discovered by any experiments, it is impossible, from this consideration to compute the effect; for no calculation of effects can be applied, when produced by causes which are subject to no law. And in the latter case, the effects of the action of one fluid upon another differ so much, in many respects, from what would be it's action as a solid body, that a computation of it's effects can by no means be deduced from the same principles. In mechanics, no equilibrium can take place between two bodies of different weights, unless the lighter acts at some mechanical advantage; but in hydrostatics, a very small weight of fluid may, without it's acting at any mechanical advantage whatever, be made to balance a weight of any magnitude. In mechanics, bodies act only in the direction of gravity; but the property which fluids have of acting equally in all directions, produces effects of such an extraordinary nature, as to surpass the power of investigation. The indefinitely small corpuscles of which a fluid is composed, probably possess the same powers, and would be subject to the same laws of motion, as bodies of finite magnitudes, could any two of them act upon each other by contact; but this is a circumstance which certainly never takes place in any of the aerial fluids, and probably not in any liquids. Under the circumstances, therefore, of an indefinite number of bodies acting upon each other by repulsive powers, or by absolute contact, under the uncertainty of the fric-

tion which may take place, and of what variation of effects may be produced by different degrees of compression, the conclusions deduced from any *theory* must be subject to considerable errors, except from *that* which is founded upon such experiments, as include in them the consequences of those principles which are liable to any degree of uncertainty.

Sir I. NEWTON seems to have been well aware of all these difficulties, and therefore, in his *Principia*, he has deduced his laws of resistance, and the principles upon which the times of emptying vessels are founded, entirely from experiment. He was too cautious to trust to theory alone, under all the uncertainties to which, he appears to have been sensible, it must be subject. He had, in a preceding part of that great work, deduced the general principles of motion, and applied them to the solution of problems which had never before been attempted; but when he came to treat of fluids, he saw it was necessary to establish his principles upon experiments; principles, not indeed mathematically true, like his general principles of motion before delivered, but, under certain limitations, sufficiently accurate for practical purposes. The principles therefore upon which we reason in this branch of philosophy, must be considered as depending upon direct experiments adapted to the particular cases, rather than upon the general principles of equilibrium and motion, as applied in Mechanics.

SECTION I.

ON THE PRESSURE OF NON-ELASTIC FLUIDS.

PROP. I.

FLUIDS press equally in all directions.

8. For if any liquid be put into a vessel, and glass tubes, straight, or bent at any angle, be put into the fluid, the fluid will rise in all the tubes to the height of the surface of the liquid in the vessel. Also, a vessel is found to empty itself in the same time through an hole at the same depth, whether the fluid spouts downwards, horizontally, upwards, or in any other direction.

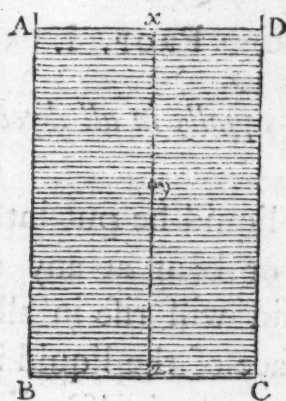
9. That the pressure upwards is equal to the pressure downwards, is one of the most extraordinary properties of fluids, and can be conceived to arise only from the perfect freedom with which the particles

move amongst each other, which freedom of motion is, probably, owing to the particles being kept at a distance by a repulsive power residing in them. This is one remarkable difference between fluids and solids, solids pressing only downwards in the direction of gravity.

PROP. II.

The pressure upon any particle of a fluid, whose density is uniform, is in proportion to it's perpendicular distance from the surface of the fluid.

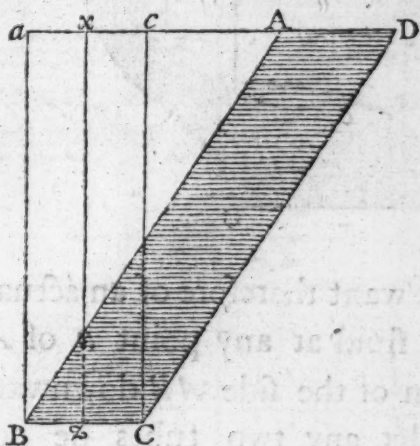
10. Case 1. Let $ABCD$ be a vessel filled with such a fluid, and draw xz perpendicular to the horizon. Now the pressure is in proportion to the weight, or number of incumbent particles; and as the density of



the fluid is uniform, and consequently all the particles are at the same distance from each other, we have $xy : xz ::$ the number of particles incumbent upon y : the number incumbent upon $z ::$ the pressure on y : the pressure on z .

Case 2. Hence if the sides of the vessel $ABCD$ be not perpendicular to the bottom, the pressure upon any

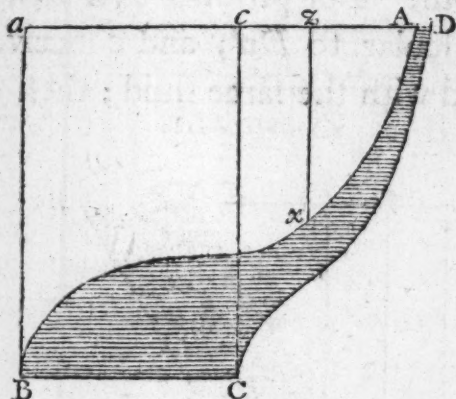
any point of BC will still be in proportion to it's perpendicular depth. For produce DA to a , and draw Ba , Cc perpendicular to Da ; and conceive $DaBC$ to be a vessel filled with the same fluid; then the pressure



on z is as zx , there being actually that perpendicular depth of the fluid incumbent upon it. Now instead of supposing the fluid $ABCD$ to be kept in it's position by the other part of the fluid AaB , conceive it to be kept in that position by the side AB of the vessel, and the effect must remain the same; for it can manifestly make no difference, by what body the fluid $ABCD$ is kept in it's place. Hence, whatever be the form of the sides, the pressure on BC will be the same as if the sides were perpendicular to the surface of the fluid, and the perpendicular height the same.

II. That the pressure at any point x is equivalent to the weight of a column xz , appears from this experiment, that if any where in AB , an hole be made perpendicular to Aa , a perfect fluid will spout upwards very nearly as far as Aa , the defect being no more than what may be supposed to arise from the resistance of

of the air, and the falling back of the upper parts of

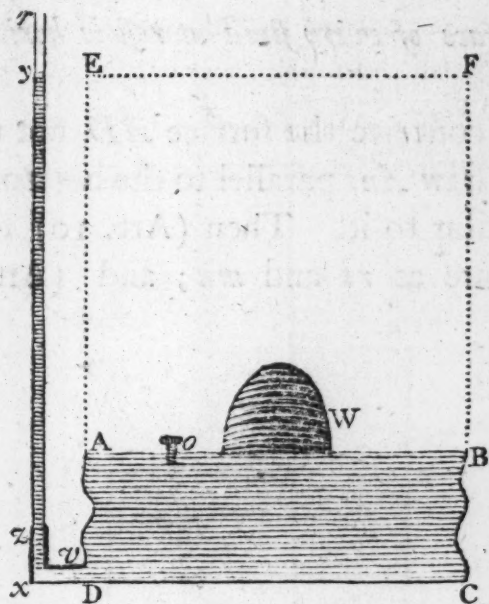


the fluid. The want therefore of an actually incumbent column xz of fluid at any point x of AB is supplied by the reaction of the side AB downwards against the fluid. Also, let any two tubes be connected, and however they may differ in magnitude or inclination, if a fluid be put into one, it will always rise to the same perpendicular altitude in the other. These experiments therefore also prove, that fluids press in proportion to their perpendicular depths.

12. Upon these two principles, that fluids press equally in all directions, and in proportion to their perpendicular depths, depends the explanation of the following experiment, called the *hydrostatical Paradox*. AB , CD are two round parallel boards, connected by leather like a pair of bellows; zxv is a brass pipe entering into the cavity, and into which the glass tube zr is fixed. Through an orifice at a a quantity of water is poured in, to keep the top and bottom at some distance from each other, and then it is stopped by a screw. Now let a very large weight W be laid upon the top AB , and the water in the tube will rise to some height xy and there it will rest, the

small

small weight of water in xy balancing the weight W ,



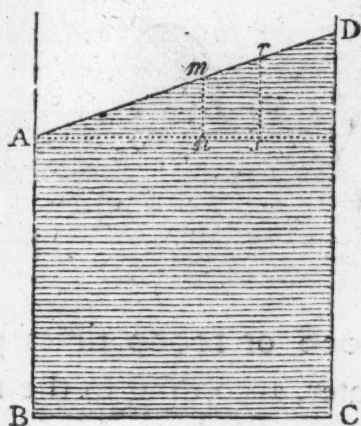
which may be 1000 or 10000 times greater than the weight of that water, according to the size of the tube; and this weight W will always be found equal to the weight of a cylinder $ABFE$ of fluid, whose surface EF is upon a level with y . The principles before mentioned will thus explain this extraordinary circumstance. The fluid at x , the bottom of the tube, is pressed with a force proportional to the perpendicular altitude xy ; this pressure is communicated horizontally in the direction xDC to all the particles, and the pressure so communicated acts equally in all directions; the pressure therefore downwards upon the bottom DC is just the same as it would be, if $DEFC$ were a cylinder of water; and it is manifest that if we take away the part $ABFE$, and place an equal weight W to act upon the other part $ABCD$, the effect will remain the same, or there will be an equilibrium.

PROP.

PROP. III.

The surface of every fluid at rest is horizontal.

13. For conceive the surface AD not to be horizontal, and draw Ans parallel to the horizon, and mn , rs perpendicular to it. Then (Art. 10) the pressures at s and n are as rs and mn ; and (Art. 8) fluids



pressing equally in all directions, the particle at n would be driven towards A ; the fluid therefore would not remain at rest. But when AB becomes horizontal, these pressures become equal, and the fluid remains at rest.

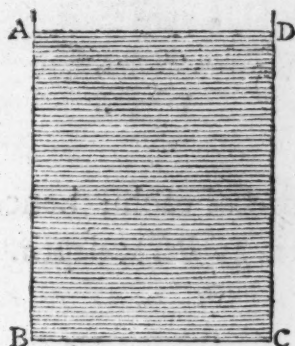
14. Cor. In like manner it appears, that if there be two fluids in the same vessel which do not mix, their common surface will be parallel to the horizon.

PROP. IV.

If the sides of a vessel $ABCD$, filled with a fluid, be perpendicular to it's bottom which is parallel to the horizon, the pressure upon the bottom will be equal to the weight of the fluid.

15. For

15. For the reaction of the sides of the vessel against the fluid, being perpendicular to the sides, or parallel to the bottom, it can neither increase nor diminish the pressure of the fluid against the bottom; also the force of gravity acting parallel to the sides, they can take off no part of the weight of the fluid, nor increase



its pressure upon BC ; the pressure therefore upon the bottom must be simply the weight of the fluid.

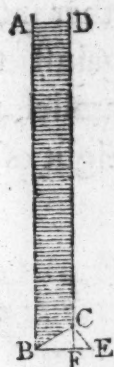
16. Cor. Hence, (Art. 36) in different vessels of this description, containing different fluids, the pressures are as the areas of the bottoms $\times$ depths $\times$ specific gravities, because the magnitude = the area of the bottom $\times$ depth.

PROP. V.

Let the bottom BC be oblique to the sides, and also so small, that the whole may be conceived to be of the same depth; then the pressure perpendicular to BC will be as $BC \times$ depth.

17. For the number of particles in contact with BC is as BC . Also, fluids press equally in all directions, and in proportion to their depths (Art. 8. 10), and the whole

whole pressure on BC must be as the number of par-



ticles $\times$ the pressure of each; hence the pressure perpendicular to BC is as $BC \times$ depth.

PROP. VI.

The pressure exerted upon BC downwards, or in the direction of gravity, is equal to the weight of the fluid.

18. Draw CE perpendicular to BC , BE to BA , and CF to BE . Now let CE represent the pressure (P) perpendicular to BC , which resolve into CF , FE , then CF is that part which acts downwards; also let p represent the pressure downwards, and π the pressure which would act upon BF as the bottom, or the weight of the fluid by Art. 15. Hence,

$$p : P :: CF : CE :: (\text{by sim. trian.}) BF : BC,$$

$$P : \pi :: BC \times \text{depth} : BF \times \text{depth} :: BC : BF.$$

$$\therefore p : \pi :: BF \times BC : BC \times BF;$$

hence $p = \pi$.

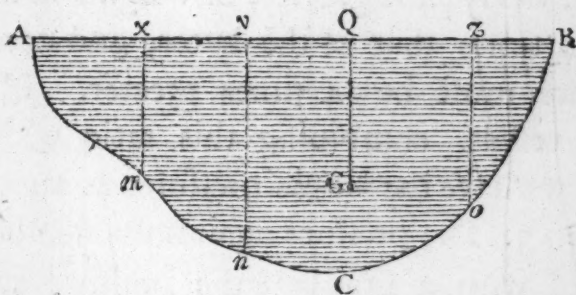
The same may be inferred directly, from the reasoning in Art. 15.

PROP.

PROP. VII.

The pressure of a fluid against any surface, in a direction perpendicular to it, varies as the area of the surface multiplied into the depth of it's center of gravity below the surface of the fluid.

19. Let ABC be a vessel, AB the surface of the fluid; divide ACB into an indefinite number of parts m, n, o , &c. let G be the center of gravity of the surface ACB , and draw GQ, mx, ny, oz , &c. perpendicular to AB . Now every part of the indefinitely small surface m may be conceived to be at the same perpendicular depth mx ; also, (Art. 10) the pressure at m is in proportion to it's depth, and that pressure is exerted equally in all directions; hence, the pressure



on m perpendicular to that surface is as $m \times mx$; consequently the whole pressure exerted on ACB , in a direction perpendicular to the surface at every point, will be as $m \times mx + n \times ny + o \times oz + \&c.$ But (by Mechanics, Art. 172.) if we consider m, n, o , &c. as representing weights in proportion to their respective magnitudes, and the whole area ACB to represent a proportional weight, then $m \times mx + n \times ny + o \times oz + \&c.$

+ &c. $= ACB \times GQ$; hence the whole pressure perpendicular to the surface varies as $ACB \times GQ$.

20. Cor. 1. The same pressure is equal to the weight of a cylinder of that fluid, the area of whose bottom is equal to the surface ACB , and altitude GQ . For the areas pressed, and the depths of the centers of gravity, are equal in the two cases, therefore the pressures are equal. But (Art. 15) the pressure on the bottom of a cylinder is equal to the weight of the fluid contained in it; consequently the pressure on the bottom ACB is equal to the same weight.

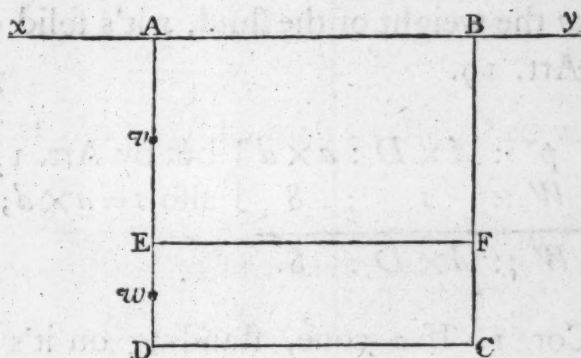
21. Cor. 2. The pressures of different fluids against different surfaces, will be as the areas $\times$ depths of their centers of gravity $\times$ specific gravities of the fluids. For it is manifest that, *cæteris paribus*, the pressures must be as the weights, or as the specific gravities (Art. 3); thus, if the same vessel be filled with mercury and water, the specific gravity, or weight, of the former will be 14 times that of the latter, and consequently the pressure must be 14 times greater. Hence, for different vessels, combining this ratio with that in Art. 19. we have the whole pressures as above.

22. Cor. 3. The pressure against the side of a cubical vessel filled with a fluid $= \frac{1}{2}$ the pressure against the bottom; for the area pressed is the same, and the depth of the center of gravity in the former case $= \frac{1}{2}$ that in the latter. Hence the pressure against the side $= \frac{1}{2}$ the weight of the fluid.

23. Cor. 4. If a cylinder, the altitude of which is a , and diameter of it's base d , be filled with a fluid; then if $p = 3,14159$ &c. the area of the base $= \frac{1}{4}pd^2$, and the area of the sides $= pda$; hence the pressure
on

on the bottom : the pressure on the side $:: \frac{1}{2}pd^2a : \frac{1}{2}pda^2 :: d : 2a$. If two equal cylinders be filled, one with mercury and the other with water, then, the pressure on the base of the former : the pressure on the sides of the latter $:: 14d : 2a :: 7d : a$.

24. Cor. 5. Let xy be the surface of a fluid, $ABCD$



a $\square$ perpendicular to it; draw EF parallel to AB , and bisect AE in v , ED in w ; then Av , Aw will be the depths of the centers of gravities of $ABFE$ and $EFCD$. Hence the pressures on these $\square$ s are as $ABFE \times Av : EFCD \times Aw :: AE \times Av : ED \times Aw :: AE \times \frac{1}{2}AE : \overline{AD - AE} \times \frac{AD + AE}{2} :: AE^2 : AD^2 - AE^2$.

PROP. VIII.

If a vessel be filled with a fluid, the pressure on any part : the whole weight of the fluid $::$ the area of that part $\times$ the depth of it's center of gravity : the solid content of the fluid.

24. By Art. 15. the pressure on the base of a cylinder filled with fluid = it's weight ; also, the weight of

the same fluid is in proportion to it's solid content. Let a vessel of any form be filled with a fluid, and let A = any part of it's surface, D = the depth of it's center of gravity, P = the pressure upon it, and W = the whole weight of the fluid, S = it's solid content; and let a cylinder, whose base = a and altitude d , be filled with the same fluid, and let p be the pressure on it's bottom, w the weight of the fluid, s it's solid content; then by Art. 19.

$$\begin{array}{l} P : p :: A \times D : a \times d \\ \text{Also, } w : W :: s : S \end{array} \left. \vphantom{\begin{array}{l} P : p :: A \times D : a \times d \\ w : W :: s : S \end{array}} \right\} \text{but by Art. 15. } w = p; \text{ hence} \\ \hline P : W :: A \times D : S.$$

25. Cor. 1. If a cone, standing on it's base, be filled with a fluid, and A = the base, we have $S = \frac{1}{3} A \times D$; consequently the pressure on the base = three times the weight of the fluid.

26. Cor. 2. If a hollow sphere be filled with a fluid, the pressure P against the whole internal surface A = three times the weight of the fluid; for the center of gravity of the surface is in the center of the sphere, whose depth D below the upper point of the fluid must therefore be equal to the radius R of the sphere, and $S = \frac{1}{3} A \times R = \frac{1}{3} A \times D$.

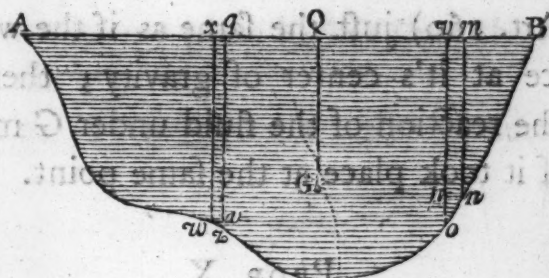
27. Hitherto we have considered the whole pressure of a fluid against any surface in a direction *perpendicular* to every point of it; but in this case, the effect on one part may partly destroy the effect on another, by their not acting in the same direction. Since we do not therefore thus get the whole joint effect in any direction, let us next consider, what is the whole pressure against any plane, in the direction of gravity.

PROP.

PROP. IX.

The pressure of a fluid downwards against the sides and bottom of any vessel, is equal to the weight of the whole fluid, provided there be, over every part of the sides and bottom, a perpendicular column of the fluid reaching to the surface

28. Let $AwzonB$ be such a vessel filled with a fluid



whose surface is AB , perpendicular to which, conceive $vmno$, $qxwz$, &c. to be indefinitely small prismatic columns into which the whole is divided; then (Art. 17.) the pressure downwards of every column is equal to the weight of the column of fluid; hence the whole pressure downwards upon the surface ACB is always equal to the whole weight of the fluid.

29. Cor. 1. As the pressure of every column downwards is equal to it's gravity, the joint effect of all the columns, or of the whole fluid, is the same as the gravity of the whole if it had been solid, and consequently (Mechanics, Art. 160) the effect is the same as if all the power was concentrated in the center of gravity.

30. Cor. 2. Hence if $ABCD$ be a vessel of fluid,
B 2
and

and acb be any part G of the fluid, it's action downwards, must be equal to the reaction of the fluid under it upwards; but the effect of G downwards, is

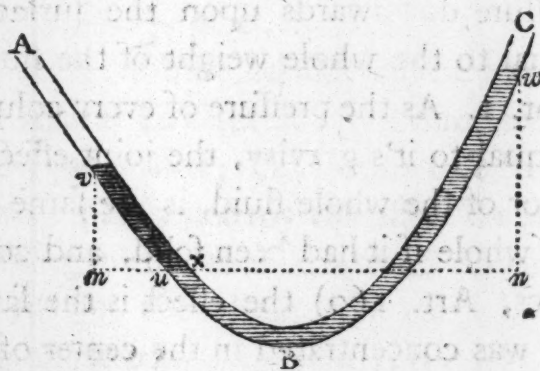


(Mech. Art. 160) just the same as if the whole effect took place at it's center of gravity; therefore the effect of the reaction of the fluid under G must be the same as if it took place at the same point.

PROP. X.

If two fluids communicate in a bent tube, their perpendicular altitudes above the plane, where they meet, are inversely as their specific gravities.

31. Let ABC be the tube, ux the plane where



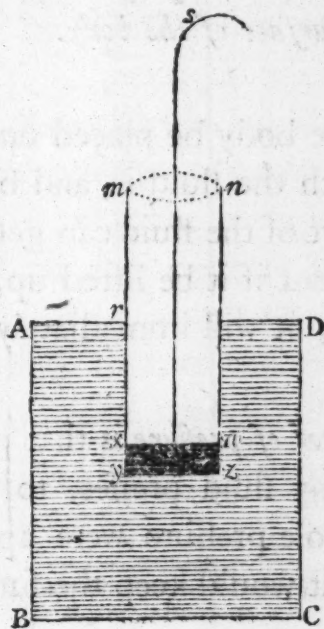
the two fluids meet, standing at v and w ; draw

$munn$

mxn parallel to the horizon, and vm , wn perpendicular to it. Let S , s represent the specific gravities of the two fluids in vu , uw ; then the area of the section ux being common to both fluids, the pressure of each fluid at that section will (Art. 21) be as it's perpendicular depth $\times$ it's specific gravity; but as the fluids are at rest, their pressures must be equal; hence $S \times vm = s \times wn$, therefore $S : s :: wn : vm$.

32. Cor. 1. Hence the same fluid will stand at the same altitude on each side; for if $S=s$, then $wn=vm$. If therefore a pipe convey a fluid from a reservoir, it can never carry it to a place higher than the surface of the fluid in the reservoir.

33. Cor. 2. If $ABCD$ be a vessel of fluid, $mxwn$



a hollow cylinder, to whose bottom a cylindrical body $wxyz$, of greater specific gravity than the fluid, may be so closely fitted, that the fluid cannot enter; then

if this body be kept in that position by the string s , and the whole be immersed perpendicularly in the vessel, until yr be to yx as the specific gravity of the body to that of the fluid, the body will remain suspended without the assistance of the string. For, we may consider the body $wxyz$ just the same as if it were a fluid of the same specific gravity, and consequently it will rest when the altitudes of the body and fluid above yz are inversely as their specific gravities.

PROP. XI.

The ascent of a body in a fluid of greater specific gravity than itself, arises from the pressure of the fluid upwards against the under surface of the body.

34. For if the body be placed upon the bottom of the vessel in which the fluid is, and be so closely fitted to it, that no part of the fluid can get under it, it will remain at rest; but if it be lifted up, so that the fluid can get under it, it will immediately rise.

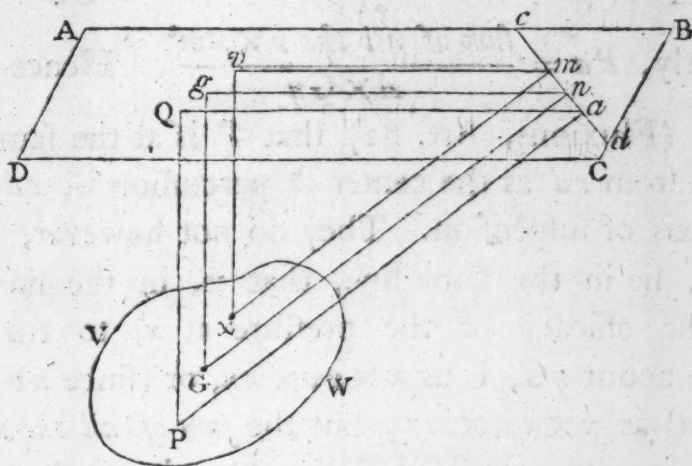
Def. The *center of pressure* is that point of a surface, against which any fluid presses, to which if a force equal to the whole pressure were applied, in a contrary direction, it would keep the surface at rest.

PROP. XII.

To find the center of pressure of a plane surface,

35. Let

35. Let $ABCD$ be the surface of the fluid, VW the plane, which being produced, let cd be it's



intersection with the surface, P the center of pressure, and G the center of gravity; and conceive the whole plane to be divided into an indefinite number of indefinitely small parts, of which one is x ; draw PQ , Gg , xv perpendicular to the surface, and Pa , Gn , xm perpendicular to cd ; and join Qa , gn , vm ; then it is manifest, that the triangles PQa , Ggn , xvm are similar. Now the pressure on x perpendicular to VW is (Art. 17.) as $x \times xv$; and (Mechanics, Art. 92) it's effect to turn the plane about cd is as $x \times xv \times xm$; but (sim. trian.) $Gn : Gg :: xm : xv =$
 $xm \times \frac{Gg}{Gn}$; hence the effect of the pressure at x to turn the plane about cd is as $x \times xm^2 \times \frac{Gg}{Gn}$; therefore the

whole effect is as the sum of all the $x \times xm^2 \times \frac{Gg}{Gn}$. But if A = the area of VW , the pressure on VW = $A \times Gg$; therefore the effect of that pressure at P , to

turn the plane about cd , is as $A \times Gn \times Pa$. Hence

$A \times Gg \times Pa = \text{sum of all the } x \times xm^2 \times \frac{Gg}{Gn}$, conse-

quently, $Pa = \frac{\text{sum of all the } x \times xm^2}{A \times Gn}$. Hence it ap-

pears (Fluxions, Art. 62) that P is at the same distance from cd as the center of percussion is, cd being the axis of suspension. They do not however, in general, lie in the same line, that is, in the line nG ; for the efficacy of the pressure at x , to turn the plane about nG , is as $x \times xv \times mn$, or (since xv varies as xm) as $x \times xm \times mn$; but the sum of all the $x \times xm \times mn$ is not generally $= 0$, therefore the whole pressure will not balance itself upon the line Gn . The situation of the line Pa must be determined, by making the sum of all the $x \times xm \times mn = 0$, which, in any particular case, may be determined by a fluxional calculation. It is not therefore true, in general, that the centers of pressure and percussion are the same point, as is asserted by writers on this subject; indeed there are but very few cases in which they coincide.



SECTION II.

ON THE SPECIFIC GRAVITIES OF BODIES.

PROP. XIII.

THE weight of a body varies as it's magnitude and specific gravity conjointly.

36. For it is manifest, that if you vary the magnitude of any body in the ratio of $M : m$, continuing it's specific gravity the same, you will alter the quantity of matter, and consequently the weight, in the same ratio. If you alter the specific gravity of the body in the ratio of $S : s$, continuing it's magnitude the same, you will alter the weight in the same ratio, (Art. 3). Hence, by the composition of ratios, if you alter both the magnitude and specific gravity, you will alter the weight in the ratio of $M \times S : m \times s$.

As the weight of a body is in proportion to it's quantity of matter, the quantity of matter is as the magnitude and specific gravity conjointly, or as the
magnitude

magnitude and density conjointly, the specific gravity and density varying in the same ratio, (Art. 5).

37. A cubic foot of rain water weighs 1000 ounces avoirdupoise, the specific gravity of which call s ; let W be the weight of another body whose magnitude in cubic feet is M , and specific gravity S . Hence, $W : 1000 :: M \times S : 1 \text{ foot} \times s$; if therefore we assume $s = 1000$ as a standard with which we may compare the specific gravities of other bodies, we have $W = M \times S$. To reduce it to the measure in cubic inches, corresponding to the specific gravity of water represented by unity, we have, $1728 : 1 :: 1000 : ,5787$ the weight of a cubic inch of rain water; hence, $W : ,5787 :: M \times S : 1 \text{ inch} \times 1(s)$, therefore, $W = ,5787 M \times S$, M being the magnitude in cubic inches, and the specific gravity of water unity. Now a troy ounce : avoirdupoise ounce :: 480 : 437,5, for an avoirdupoise ounce contains 437,5 grains troy. Hence, to reduce W to troy weight, as the troy ounce is the greater, the weight expressed in troy ounces will be less in the same proportion; therefore, $480 : 437,5 :: ,5787 M \times S : ,52746 M \times S = W$ the weight in troy ounces, $= 253,18 M \times S$ grains. If $M = 1$, $S = 1$, this gives 253,18 grains for the weight of a cubic inch of rain water*.

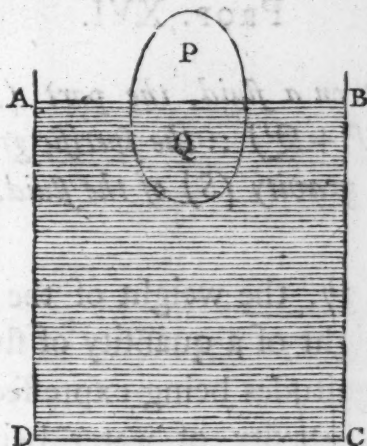
* Hence, we may very accurately determine the diameter d of a sphere whose specific gravity is s , that of water being unity. For the content of a sphere whose diameter $= 1$ is 0,5236; therefore, $1 : 0,5236 :: 253,18 \text{ grains (the weight of 1 cubic inch)} : 132,428 \text{ grains, the weight of a sphere of water whose diameter is 1 inch.}$ Hence, since the weights are as the magnitudes and specific gravities conjointly, and the magnitudes of spheres are as the cubes of their diameters, we have $132,428 s d^3 = w$, consequently, $d = \sqrt[3]{\frac{w}{s}} \times ,19612$.

PROP.

PROP. XIV.

If a body float on a fluid, it displaces as much of the fluid as is equal to the weight of the body.

38. For the body $P + Q$ is supported by the pressure of the fluid upwards against the part immersed; also,



the same pressure supported a quantity of fluid equal in bulk to Q , before the body was immersed, that space being occupied by the fluid; and as the same pressure must sustain the same weight, when there is an equilibrium, the weight of the body must be equal to the weight of a quantity of fluid equal in bulk to Q .

PROP. XV.

If a body float on a fluid, the centers of gravity of the body and of the fluid displaced, must, when the body is at rest, be in the same vertical line.

39. For

39. For (Art. 30) the effect of the pressure of the fluid upwards is the same as if the whole took place at the center of gravity of the fluid displaced; if therefore this action of the fluid upwards against the body in a vertical line do not pass through the center of gravity of the body, it must (Mech. Art. 164) give the body a rotatory motion.

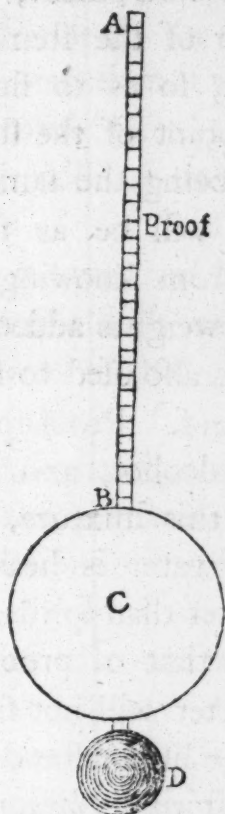
PROP. XVI.

If a body float on a fluid, the part ($\mathcal{Q}$) immersed : the whole body ($P + \mathcal{Q}$) :: the specific gravity (s) of the body : the specific gravity (S) of the fluid.

40. By Art. 37. the weight of the body is $\overline{P + \mathcal{Q}} \times s$, and the weight of a quantity of fluid equal to $\mathcal{Q}$ is $\mathcal{Q} \times S$, the magnitudes being expressed in cubic feet, and the specific gravity of water being 1000; but (Art. 39) their weights are equal; hence, $\overline{P + \mathcal{Q}} \times s = \mathcal{Q} \times S$; consequently, $\mathcal{Q} : P + \mathcal{Q} :: s : S$. We here suppose the part P to be in vacuo. But when bodies float upon a fluid, the part P being in the air, this rule will not be accurately true; near enough so, however, for all practical purposes. The effect of the air, if necessary, may be computed from Art. 52.

41. Cor. Hence, if the same body float upon two different fluids, the parts immersed will be inversely as the specific gravities of the fluids; for in this case $P + \mathcal{Q}$ and s being constant, $\mathcal{Q}$ varies inversely as S . Upon this principle is constructed the **HYDROMETER**, an instrument for measuring the specific gravities of such fluids, as do not differ much in their specific gravities.

gravities. *AB* is a graduated stem fixed to a hollow globe *C*, which is annexed to another sphere *D*, into which mercury or shot is put, in order to make the instrument sink in the fluid, and keep it vertical. Now let us suppose the whole bulk of the instrument to be



represented by 4000, and each of the divisions by 1 of such parts, and let the whole length of the stem contain 50 of such parts. Now if this instrument be put into one fluid, and it sinks to 30, and in another, it sinks to 20, then the parts immersed will be 3970 and 3980 respectively; and the specific gravities of these two fluids, being inversely as the parts immersed, will be as 3980 : 3970. This can only be applied to those fluids which come within the extent of the scale. This instrument

instrument therefore generally consists of two stems, one of which can be taken off and the other put on, the stems being adapted to fluids of different specific gravities, one measuring what the other will not.

Mr. NICHOLSON has lately made a considerable improvement in this instrument, by placing a small brass cup on the top of the stem, into which small weights may be put, so as to sink it into different fluids to the *same* point of the stem. In this case, the part immersed being the same, the specific gravities of the fluids will be as the whole weights, which are known, from knowing the weight of the instrument, and the weights added.

This instrument is also used to find whether a fluid is above or below proof. Proof spirits consist, half of pure spirits, called alcohol, and half of water; the instrument put into this mixture, sinks to *Proof* upon the scale. Now as water is heavier than the spirits, if there be more water than spirits, the specific gravity will be greater than that of proof spirits, and consequently the hydrometer will not sink so far as proof, or the surface of the liquor stands below proof, and the strength of the spirits is *below* proof: but if there be less water than spirits, the specific gravity will be less than that of proof spirits, and therefore the hydrometer will sink below proof, or the surface of the fluid will stand above proof, and the spirits are *above* proof. This is the instrument which the Officers of Excise generally use when they examine liquors, in order to determine their strength, for the purpose of ascertaining the duty.

PROP.

PROP. XVII.

The weight which a body loses when wholly immersed in a fluid, is equal to the weight of an equal bulk of the fluid.

42. First, let the body be of the same specific gravity as the fluid, and then it is manifest that it will remain at rest, because (Art. 3) it is of the same weight as an equal bulk of the fluid, and therefore the pressure upwards of the fluid beneath will support the body equally as it supported the fluid which occupied the space before the body was put in; in this case, the body is said to have lost all its weight, or the weight of an equal bulk of fluid. Now let the specific gravity, and consequently the weight of the body, be increased, then the pressure of the fluid upwards against the body still continuing the same, that action must still take off the same weight from the body, that is, the weight of an equal bulk of fluid.

When we say, that a body loses part of its weight in a fluid, we do not mean that its absolute weight is less than it was before, but that it is partly supported by the reaction of the fluid under it, so that it requires a less power to support it.

43. Cor. 1. Hence, when a body is weighed in air, in order to get its absolute weight, we must add to it the weight of an equal bulk of air. If, for example, a body, whose magnitude is one cubic foot, weigh 1500 ounces troy in air, we must add 21 pennyweights to it, which is the weight of a cubic foot of air, and it gives 1500 oz. 21 dwts. the real weight of the body, or its weight in vacuo.

44. Cor.

44. Cor. 2. Hence also it follows, that if two bodies of the same weight in air, be put into a denser fluid, the smaller body will preponderate, since it loses a less weight than the other. And if they weigh equally in any fluid, and then be brought into a rarer medium, the greater will preponderate, because having lost more weight in the denser fluid than the other body, when carried into a rarer fluid it will regain more weight, and therefore will weigh most in the lighter fluid.

PROP. XVIII.

A body immersed in a fluid, ascends or descends with a force equal to the difference between it's own weight and the weight of an equal bulk of fluid, the resistance of the fluid not being considered.

45. Let W be the weight of the body, w the weight of an equal bulk of the fluid. Now we may consider the body as descending by it's own weight W , and (Art. 42) as opposed in it's descent by w ; hence, when W is greater than w the body descends, and when W is less than w it must ascend, and, in both cases, by the difference between W and w , as they oppose each other.

46. Cor. Let the specific gravity of the fluid : that of the body :: 1 : b ; then, (Art. 3) $w : W ::$

1 : b ; hence $w = \frac{W}{b}$, consequently the relative gravity,

or weight of the body in the fluid, $= W - \frac{W}{b}$.

PROP.

PROP. XIX.

The weight (w) which a body loses when immersed in a fluid : it's whole weight (W) :: the specific gravity of the fluid (s) : the specific gravity of the body (S).

46. For (Art. 43) w is the weight of a bulk of fluid equal to the bulk of the body, the weight of which is W ; but (Art. 3) the weights of equal bulks are as their specific gravities; consequently $w : W :: s : S$.

Ex. If a body weigh 12 lb. in air, and 7 lb. in water, then 5 lb. is the weight lost; hence, $5 : 12 ::$ the specific gravity of water : the specific gravity of the body. Thus we compare the specific gravities of bodies and fluids of less specific gravities.

47. Cor. 1. Hence, $S = \frac{s \times W}{w}$; if therefore s be given, that is, if different bodies be weighed in the same fluid, then will S vary as $\frac{W}{w}$.

Ex. If a body A weigh 9 lb. in air, and 7 lb. in any fluid, and another body B weigh 13 lb. in air, and 6 lb. in the same fluid, then the specific gravity of A : that of $B :: \frac{9}{2} : \frac{13}{7} :: 63 : 26$. Thus we compare the specific gravities of two bodies.

48. Cor. 2. Hence also, $s = \frac{S \times w}{W}$; if therefore S and W be given, that is, if the same body be weighed in different fluids, then will s vary as w .

Ex. If a body lose 6 lb. in one fluid, and 5 in another, the specific gravities of these fluids are as 6 : 5. Thus we compare the specific gravities of two fluids.

If one of the fluids be mercury, the body must be either gold or platina, these being the only two metals which will sink in mercury. It is better, however, in this case, to compare the mercury with water (Art. 46), weighing the mercury in the water by putting it into a small glass vessel suspended from the scale, first balancing the vessel in the water.

49. The effect of the air in diminishing the weight of a body is not here considered. If the body which is weighed in the fluid be wood, it should first be well rubbed over with grease, or varnished, to prevent it from imbibing any of the fluid.

50. Since the specific gravities of fluids vary when their temperatures vary, in comparing the specific gravities of different fluids, we must first reduce them to some one temperature, as a standard. This temperature is arbitrary; and it must be observed that, from the *different* expansions of fluids for the *same* variation of temperature, the proportion of the specific gravities at different degrees of temperature will be different. Many solid bodies are also subject to a variation of their specific gravities, from the variation of their temperatures.

PROP. XX.

To find the specific gravity of a body Q, which is lighter than the fluid in which it is weighed.

51. Connect

51. Connect $\mathcal{Q}$ with another body H which is heavier than the fluid, so that together they may sink; let the weight of P in the fluid be a , the weight of $P + \mathcal{Q}$ in the fluid be b , the weight of $\mathcal{Q}$ out of the fluid be d , and the weight of a bulk of fluid equal to $\mathcal{Q}$ be e . Now, as $\mathcal{Q}$ is of less specific gravity than the fluid, it will (Art. 45) ascend with the force $e - d$. Also, as $\mathcal{Q}$ by itself would ascend, when it is connected with P , they will together have a less descending power in the fluid than P of itself would have, by the power of $\mathcal{Q}$'s ascent; now the weight or descending power of P , in the fluid is a , and the weight or descending power of $P + \mathcal{Q}$ in the fluid is b , therefore the difference $a - b$ gives the ascending power of $\mathcal{Q}$; hence, $e - d = a - b$, and $e = a - b + d$; but (Art. 3) the weights of equal bulks are as the specific gravities; hence, the specific gravity of $\mathcal{Q}$: the specific gravity of the fluid $:: d : a - b + d$.

PROP. XXI.

If a lighter fluid rest upon a heavier, and their specific gravities be as $a : b$, and a body, whose specific gravity is c , rest with one part P in the upper fluid, and the other part $\mathcal{Q}$ in the lower, then $P : \mathcal{Q} :: b - c : c - a$.

52. The body will rest when it has displaced as much of the two fluids as is equal in weight to itself, for the reason given in Art. 38. Now (Art. 37) the weight of the body is $c \times \overline{P + \mathcal{Q}}$; also, the weight of the lower fluid displaced is $b \times \mathcal{Q}$, and of the upper fluid, $a \times P$; hence, $a \times P + b \times \mathcal{Q} =$

$c \times 2$
 $c \times \overline{P + \mathcal{Q}}$

$c \times \overline{P + Q} = c \times P + c \times Q$; therefore, $b \times Q - c \times Q =$



$c \times P - a \times P$, or $\overline{b - c} \times Q = \overline{c - a} \times P$; hence, $P : Q :: b - c : c - a$.

53. Cor. Hence, $Q : P + Q :: c - a : b - a$, and if a be so small that it may be neglected, then $Q : P + Q :: c : b$, as in Art. 40.

PROP. XXII.

If a and b be the specific gravities of two fluids to be mixed together, P and Q their magnitudes, and c the specific gravity of the compound, then $P : Q :: b - c : c - a$.

54. By Art. 37. the weight of P is $a \times P$, the weight of Q is $b \times Q$, and the weight of the compound is $c \times \overline{P + Q}$; but the weight of the compound must be equal to the sum of the weights of the two parts; hence (as in Art. 52) $P : Q :: b - c : c - a$.

55. It is here supposed, that the magnitude $P + Q$, of the compound, is equal to the sum of the magnitudes of the two parts when separate. But it very often happens,

pens, that the magnitude of the mixture is less than this sum, owing, probably, partly to the constituent particles of the different fluids being of different magnitudes, and partly to their chemical affinity. This is called a *penetration of dimensions*. Thus, for instance, if a pint of water and a pint of oil of vitriol be mixed together, the mixture will not make a quart. The specific gravity of the compound is manifestly increased by this circumstance.

Ex. Let the specific gravity of gold be 19, of silver 11, and of the compound 14; then the magnitude of the silver in the mixture : the magnitude of the gold :: $19 - 14 : 14 - 11 :: 5 : 3$.

A certain quantity of gold having been given by King HIERO to make him a crown, the artists secreted part of the gold, and substituted the same weight of silver. This being suspected, ARCHIMEDES was employed to discover the cheat; but it is not related in what manner he did it, except that by going into a bath, the rising of the water suggested to him the method of finding the magnitudes of irregular bodies.

56. If we want to find the proportion of the *weights* of each body, we must take the ratios of their magnitudes, and of their respective specific gravities conjointly; hence, the weights of *P* and *Q* are as $a \times \overline{b - c} : b \times \overline{c - a}$. In the above example, therefore, the weight of the silver : the weight of the gold :: $11 \times 5 : 19 \times 3 :: 55 : 57$.

TABLE OF SPECIFIC GRAVITIES.

Refined gold	19637
English guinea	17793
Mercury	14019
Lead	11325
Refined silver	11087
Standard silver	10535
Bismuth	9700
Copper of Japan	9000
Copper of Sweden	8843
Hammered brass	8349
Cast brass	8100
Turbeth mineral	8235
Cinnabar, factitious	8200
Cinnabar, natural	7300
Elastic steel	7820
Soft steel	7738
Iron	7645
Pure tin	7471
Glass of antimony	5280
A pseudo topaz	4270
A diamond	3400
Crystal glass	3150
Island crystal	2720
Rock crystal	2658
Common glass	2620
Fine marble	2704

Stone

Stone of a mean gravity	. 2500
Selenitis	2252
Sal gemmæ	2143
Nitre	1900
Alabaster	1875
Dry ivory	1825
Brimstone	1800
Dantzic vitriol	1715
Allum	1714
Borax	1714
Calculus humanus	1700
Oil of vitriol	1700
Oil of tartar	1550
Bezoar	1500
Honey	1450
Gum arabic	1375
Spirit of nitre	1315
Aqua fortis	1300
Pitch	1150
Spirit of salt	1130
Craffamen of the human blood	1126
Spirit of urine	1120
Human blood	1054
Amber	1030
Serum of human blood	1030
Milk	1030
Urine	1030
Dry box-wood	1030
Sea-water	1030
Common water	1000
Camphire	996
Bees wax	955
Linseed oil	932

Dry oak		925
Oil olive		913
Spirit of turpentine		864
Rectified spirit of wine		866
Dry ash		800
Dry maple		755
Dry elm		600
Dry fir		550
Cork		240
Air		$1\frac{1}{3}$

The specific gravity of rain water being here represented by 1000, and a cubic foot of rain water weighing 1000 ounces avoirdupoise, the numbers against each substance represents the weight of a cubic foot thereof in avoirdupoise ounces. The specific gravities are subject to a small degree of variation, arising from the variation of temperature of the air.

The scales which are made use of to weigh bodies, in order to determine their specific gravities, are called the *Hydrostatic Balance*.



SECTION III.

ON THE RESISTANCE OF FLUIDS.

57. **T**HE resistance of a body moving in a fluid arises from the inertia, the tenacity, and friction of the fluid, admitting the particles to be in contact. The latter cause, granting it to exist, is probably very small; and the second is, in most fluids, inconsiderable when compared with the inertia. The resistance therefore, which we shall here consider, is that arising from the inertia of the fluid.

PROP. XXIII.

If a plane surface move in a fluid with a velocity V , in a direction perpendicular to it's plane, the resistance, within certain limits of the velocity, varies as V^2 .

58. For the resistance must vary as the number of particles which strike the plane in a given time, multiplied

plied into the force of each against the plane. Now the number of particles which the plane strikes in a given time must evidently be in proportion to V ; also, the force of each particle is as V , and as action and reaction are equal and contrary, the reaction of every particle of the fluid against the plane must be as V ; hence the resistance varies as $V \times V$, or as V^2 . This is found, by experiment, to be very nearly true, when the velocity is small.

59. This proof supposes, that after the plane strikes a particle, the action of that particle immediately ceases, and the particle itself to be, as it were, annihilated; but the particles, after they are struck, must necessarily be made to diverge and act upon the particles behind, which makes some difference between this theory and experiment. Also, by increasing the velocity of the body, the action of the fluid behind it, to impel it in the direction of it's motion, will be diminished, and consequently the retardation will, on this account, be increased. Mr. ROBINS found, from experiment, that when a bullet moves with the velocity of sound, or with a greater velocity (in which case, a vacuum is left behind the body, and the pressure forwards from behind then ceases), that the resistance is always greater than this law gives it. When bodies descend in fluids, such as water, the resistance is very nearly as V^2 , because the body can never acquire a velocity beyond a certain limit. We will therefore, in the articles here given upon resistances, suppose the resistance to vary as V^2 . This law of resistance was established by Sir I. NEWTON, from a variety of experiments; see the *Principia*, Vol. II. Prop. 31, Scholium; also Mr. PARKINSON's *Hydrostatics*, page 26.

PROP.

PROP. XXIV.

When different planes move in directions perpendicular to their surfaces, in different fluids, and with different velocities, the resistances will be as the squares of their velocities $\times$ the densities of the fluids $\times$ the areas of the planes.

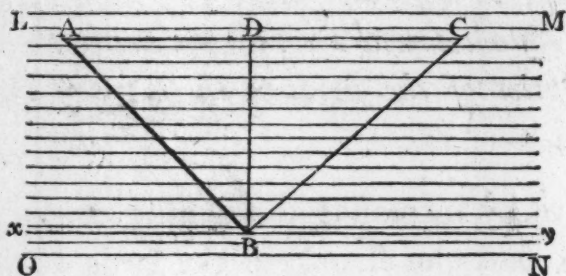
60. For by increasing the density of the fluid, the number of particles struck in the same time will be greater in the same proportion, and consequently the resistance will, *cæteris paribus*, be greater in the same ratio. Also, by increasing the area of the plane, the greater will be the number of particles struck in the same ratio, and therefore the resistance will be greater in the same proportion. And (Art. 58) when the velocities vary, *cæteris paribus*, the resistance varies as V^2 . Hence, combining these ratios together, the resistance will be as $V^2 \times$ the densities of the fluids $\times$ areas of the planes.

PROP. XXV.

If a plane move obliquely in a resisting medium, with an uniform velocity, and after the resolution of the force with which the plane strikes the fluid, the whole of that part which acts perpendicular to the plane takes effect, the resistance perpendicular to the plane will vary as the square of the sine of the angle of inclination.

61 Let AB be the plane moving in the medium $LMNO$ in the direction xy ; draw AC parallel to xy , meeting BC perpendicular to AB in C ; and let BD be perpendicular

perpendicular to AC . Now the quantity of fluid which AB has to oppose by it's motion, being that which is contained between AC and xy , is manifestly in pro-



portion to BD , or to the sine of BAC , because $AB : BD :: \text{rad.} = 1 : \sin. BAD$ or BAC , and the first and third terms being constant, the second varies as the fourth. Also, as the plane acts against the fluid at the angle CAB , let AC be taken to represent the whole force of the plane acting against the fluid upon supposition that no part thereof was lost, which force would be constant, the velocity of the plane being uniform; then, by the resolution of motion, the force acting perpendicular to the plane will be in proportion to BC , or to the sine of BAC , for $AC : BC :: \text{rad.} = 1 : \sin. BAC$, where the first and third terms being constant, the second varies as the fourth. Hence, the whole action of the plane against the fluid in the direction BC (being in proportion to the whole quantity of fluid which opposes it's motion, and it's effect in the direction BC conjointly) will be as $\sin. BAC \times \sin. BAC$, or as $\sin. BAC^2$. And as action and reaction are equal and contrary, the action of the fluid against the plane in the direction CB , or the resistance of the fluid, must vary in the same ratio.

PROP.

PROP. XXVI.

The resistance of the same fluid to oppose the plane in the direction of it's motion varies as $\overline{\sin. BAC}^3$, supposing, after the resolution of the reaction of the fluid in the direction CA, into CB and BA, the part BA to be entirely lost, and CB to take effect.

62. As the whole effect of the resistance of the fluid upon the plane is that part which is perpendicular to it, let CB represent that whole resistance, and resolve it into CD, DB, then will CD represent the resistance which opposes the motion of the body; now, $\text{rad.} = 1 : \sin. DBC$, or $\sin. BAC$, $:: CB : CD = CB \times \sin. BAC$; but CB, as representing the whole resistance in the direction CB, varies as $\overline{\sin. BAC}^2$ (Art. 61); hence, CD varies as $\overline{\sin. BAC}^3$.

By experiments on plane bodies moving both in air and water, I find that they are not resisted according to the laws here deduced. Part of the difference may probably be owing to the two latter causes mentioned in Art. 57, but it principally arises from the force, after resolution, not taking effect as here supposed, that part which is parallel to the plane not being all lost. But the further consideration of this subject falls not within the plan of this work, which is intended only to be an elementary treatise.

PROP. XXVII.

The same supposition being made, the resistance of the plane, in a direction perpendicular to that of it's motion, varies as $\overline{\sin. BAC}^2 \times \cos. BAC$,

63. For

63. For by the last Art. DB will represent that part of the whole resistance which acts perpendicularly to the direction of the motion; hence, $\text{rad.} = 1 : BCD$, or $\text{cof. } BAC$, $:: CB : DB = CB \times \text{cof. } BAC$; but CB , as representing the whole effective part of the force, varies as $\overline{\text{fin. } BAC}^2$ (Art. 61); therefore, DB varies as $\overline{\text{fin. } BAC}^2 \times \text{cof. } BAC$.

64. If instead of supposing the plane to move in the fluid, we suppose the plane to be at rest and the fluid to move against it, the action of the fluid against the plane will be just the same as it's reaction when the plane moves. Hence, the last article will show the effect which the wind has upon the sails of a wind-mill, when at rest, to put them in motion, admitting our hypothesis, respecting the efficacious part of the force, to be true.

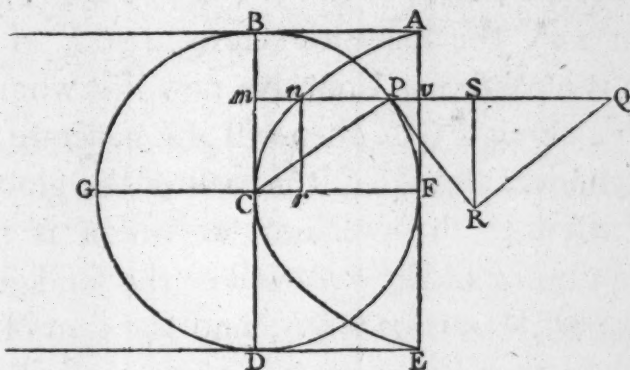
65. If, in the three last articles, the area of the plane, the velocity and density of the fluid be not given, then (for the reasons in Arts. 58, 60) the resistance will vary in the above ratios, and the area of the plane, square of the velocity and density conjointly.

PROP. XXVIII.

The same supposition being made, let a cylinder, moving in the direction of it's axis, and a sphere of the same diameter, move in the same fluid with the same velocity; then will the resistance to the motion of the cylinder be double to that of the globe.

66. Let AFE be a diameter of the end of a cylinder, parallel and equal to BD a diameter of the sphere $BFDG$ whose center is C , and CF , DE , BA perpendicular

dicular to AE ; draw QP parallel to CF the axis of the cylinder, and let it represent the force with which a particle of fluid would act perpendicularly at v , the end of the cylinder, in which case no part is lost. Now conceiving the the same particle to act upon



the globe at P , part of it's effect will be lost by the obliquity of the stroke; draw PR a tangent to the sphere in the plane PCF , and QR perpendicular to it; draw also RS perpendicular to QP , and produce QP to m . Resolve the whole force QP into RP , QR , then we here suppose that the part RP to be wholly lost, and the part QR only to be effective; resolve this into QS , SR , then QS is employed in opposing the motion of the globe, and SR (being perpendicular to it) can have no effect in that respect, and moreover it will be destroyed by an equal and opposite force, acting at a point, equidistant from F , on the other side. Hence the force with which the point v at the end of the cylinder is retarded: the force with which the corresponding point P on the globe is retarded $:: QP : QS ::$ (because $QP : QR :: QR : QS$) $QP^2 : QR^2 ::$ (by sim. trian. PRQ , PMQ) $PC^2 : Pm^2 :: PC : \frac{Pm^2}{PC} ::$ (taking $vn = Pm^2$)

$\frac{Pm^2}{PC}$) $PC : vn :: vm : vn$; consequently the whole resistance on the cylinder : that on the globe :: the sum of all the vm 's : the sum of all the vn 's. Draw nr parallel to mC . Now $vm : vn :: PC^2 : Pm^2$, therefore vm , or CF , : mn , or Cr , :: PC^2 , or AF^2 , : $PC^2 - Cm^2$, or rn^2 , the locus therefore $AnCE$ of all the points n is a parabola. Conceive now this whole figure to revolve about FCG , then will AE generate the end of the cylinder, and BFD that half of the globe which opposes itself to the medium, or which is resisted; also, the sum of all the vm 's will be the solid generated by the parallelogram $AEDB$, and the sum of all the vn 's will be the solid generated by the inscribed parabola ACE , which solids (Fluxions, pag. 78) are as 2 : 1; hence, the resistance of the cylinder : the resistance of the globe :: 2 : 1.

It appears by experiment, that this proposition is not true when bodies move either in air or water, the resistance of the globe, compared with that of the cylinder, being less than that which the theory gives it.

PROP. XXIX.

The same supposition being admitted, if a globe whose diameter is d move in a resisting medium whose density is n , with a velocity v , the resistance will vary as $v^2 d^2 n$.

67. For the resistance of a globe is (Art. 66) equal to half the resistance of the base of a cylinder of the same diameter, moving in the direction of it's axis with the same velocity; therefore the resistance
of

of the globe varies as the resistance of the cylinder; but the end of the cylinder being a circle, whose area is as d^2 , the resistance (Art. 60) varies as $v^2 d^2 n$; therefore the resistance of the globe varies as $v^2 d^2 n$.

If the Reader wish to see any thing further upon the resistance of bodies moving in fluids, he may consult the *Fluxions*, page 125.

PROP. XXX.

As a body descends in a fluid, it continually adds more weight to the fluid until it has acquired it's greatest velocity, at which time, the weight added to the fluid from the resistance is equal to the relative weight of the body.

68. For as long as the velocity of the body increases, the action of the body upon the fluid will continue to increase; and when the body has acquired it's greatest velocity, the resistance becomes equal to the weight of the body in the fluid, and the body then acts against the fluid with it's whole relative weight.



SECTION IV.

ON THE TIMES OF EMPTYING VESSELS, AND ON SPOUTING FLUIDS.

PROP. XXXI.

IF a fluid run through any tube, kept continually full, and the velocity of the fluid in every part of the same section be the same, the velocities in different sections will be inversely as the areas of the sections.

69. For the same quantity of fluid runs through every section in the same time; now the quantity running through any section (A) with the velocity (V), in any given time, is manifestly in proportion to A and V conjointly, or to $A \times V$; and as this quantity is constant, $A \times V$ is constant, consequently V varies inversely as A .

70. Cor. Hence, the velocity of water in a river increases as the breadth and depth decrease, the rule however cannot be applied with accuracy here, as the water at the bottom, from it's unevenness, cannot
move

move with the same velocity as at the top ; and where the rise and fall at the bottom is very quick, there is probably a great deal of stagnant water.

PROP. XXXII.

Let a vessel be kept filled with a fluid whilst it continues to run out at an orifice ; and let the quantity run out in a given time, and the area of the orifice be given, to find the velocity at the orifice.

71. Let a = the area of the orifice, m = the quantity run out in the time t'' , and v = the velocity. Conceive all the fluid which runs out to form a cylinder whose base is a , and length l ; then $a \times l = m$, hence $l = \frac{m}{a}$; therefore, in the time t'' the fluid, with the first velocity v , would have described the space $\frac{m}{a}$; hence, $t'' : 1'' :: \frac{m}{a} : v = \frac{m}{at}$ the velocity in a second at the orifice.

PROP. XXXIII.

If a fluid run out from the bottom or side of a vessel, and the area of the orifice be very small when compared with the bottom, the velocity at the orifice is that which a body would acquire in falling through a space equal to half the altitude of the fluid above the orifice, very nearly.

72. When a fluid issues from a vessel, the water rushing towards the orifice in all directions causes a contraction in the stream; and at a distance from the orifice equal to it's diameter, Sir I. NEWTON measured

the diameter of the section of the stream (which section he called the *vena contracta*), and found it to be to the diameter of the orifice as $21 : 25$. Now the area of the orifice : the area of the vena contracta (they being supposed to be similar) $:: 25^2 : 21^2$, which is very nearly as $\sqrt{2} : 1$; hence, as (Art. 69) the velocity is inversely as the area of the section, the velocity at the vena contracta : the velocity at the orifice $:: \sqrt{2} : 1$. Also from the quantity of water running out in a given time, and the area of the vena contracta, Sir I. NEWTON found (Art. 71) that the velocity at the vena contracta is that which a body acquires in falling down the altitude of the fluid above the orifice; hence, the velocity at the orifice (being less than that at the vena contracta in the ratio of $\sqrt{2} : 1$) is (Mech. Art. 241) that which a body would acquire in falling down half the altitude.

73. The principle to be established, in order to determine the time of emptying a vessel through an orifice, is the relation between the velocity of the fluid at the orifice, and the altitude of the fluid above it. Most writers upon this subject have considered the column of fluid over the orifice as the expelling force, and from thence some have found the velocity at the orifice to be that which a body would acquire in falling down the *whole* depth of the fluid; and others, that it is such as is acquired in falling through *half* the depth; and this without regard to the magnitude of the orifice; whereas it is manifest from experiment, that the velocity at the orifice, the depth of the fluid being the same, depends upon the proportion which the magnitude of the orifice bears
to

to the magnitude of the bottom of the vessel. Conclusions thus contrary to matter of fact show, either that the principle assumed is not true, or that it is not applicable to the present case. The most celebrated theories upon this subject are those of D. BERNOULLI and M. D'ALEMBERT; the former deduced his conclusions from the principle of the *conservatio virium vivarum*, or, as he calls it, the *equalitas inter descensum actualem ascensumque potentialem*, where by the *descensus actualis* he means the actual descent of the center of gravity, and by the *ascensus potentialis* he means the ascent of the center of gravity, if the fluid which flows out could have it's motion directed upwards; and the latter, from the principle of the *equilibrium* of the fluid. This principle of M. D'ALEMBERT leads immediately to that assumed by D. BERNOULLI, and consequently they both obtain the same fluxional equation, the fluent of which expresses the relation between the velocity of the fluid at the orifice, and the perpendicular altitude of the fluid above it. How far the principles here assumed can be applied in our reasoning upon fluids, can only be determined by comparing the conclusions deduced from them with experiments.

The general fluxional equation above mentioned cannot be integrated, and therefore the relation between the velocity of the fluid at the orifice and it's depth cannot from thence be determined in all cases. If the magnitude of the orifice be indefinitely less than that of the surface of the fluid, the equation gives the velocity of the fluid equal to that which a body would acquire by falling *in vacuo* through a space equal to the depth of the fluid. But the velo-

city here determined is not that at the orifice, but at the vena contracta; for the fluid by flowing in all directions to the orifice contracts the stream, and the velocity being inversely as the area of the section, the velocity continues to increase as long as the stream, by the expelling force of the fluid, continues to decrease, and when the stream ceases to be contracted by that force, at that section of the stream, or at the vena contracta; the velocity is found, by this theory, to be that which a body would acquire in falling through a space equal to the depth of the fluid. To determine therefore, by theory, the time in which a vessel empties itself, we must know the proportion between the area of the orifice and the area of the vena contracta; but no theory will give this. The times therefore of emptying vessels, even in the most simple case, cannot be determined by theory alone.

PROP. XXXIV.

If a vessel empty itself through a very small orifice, the velocity of the fluid at the orifice varies as the square root of the altitude (a) of the fluid above it.

74. For the velocity at the orifice is that which is acquired (Art. 72) in falling down $\frac{1}{2}a$, and consequently (Mech. Art. 241) it varies as $\sqrt{\frac{1}{2}a}$, or as $\sqrt{a}$.

PROP. XXXV.

If a vessel empty itself through an orifice at the bottom, and the area of the section, parallel to the bottom, be every where the same, the velocity of the surface of the fluid is uniformly retarded.

75. For

75. For (Art. 69) the velocity of the descending surface is to the velocity at the orifice as the area of the orifice to the area of the surface, which is a constant ratio; hence, the velocity of the descending surface varies as the velocity at the orifice, or as $\sqrt{a}$ by the last article; that is, the velocity of the descending surface varies as the square root of the space which it has to describe, which is exactly the case of a body projected perpendicularly from the Earth's surface, where (Mech. Art. 248) the velocity is as the square root of the space to be described; and as the retarding force is constant in the latter case, it must also be constant in the former.

PROP. XXXVI.

If a cylindrical or prismatic vessel, having an orifice at the bottom, be kept constantly full, twice the quantity which the vessel contains will run out in the time it would have emptied itself.

76. For the surface of the fluid being uniformly retarded, and it's velocity becoming equal to nothing at the bottom, the space (Mech. Art. 237) which the surface *would* describe with the first velocity continued uniform for the time in which the vessel would empty itself is double the space which the surface actually *does* describe in the time it empties itself; in that time therefore the quantity discharged in the former case is double that in the latter, because the quantity run out when the vessel is kept full may be measured by what *would* be the descent of the surface, if it could descend with it's first velocity.

PROP. XXXVII.

If a cylindrical or prismatic vessel, whose altitude is h , empty itself through a very small orifice (a) at the bottom (A), the time (t) of emptying itself = $,3526 \times \frac{A}{a} \times \sqrt{h}$.

77. By the last Proposition, in the time t the quantity discharged with the first velocity (v) is equal to $2A \times h$; hence, (Art. 71.) $v = \frac{2A \times h}{a \times t}$; therefore,

$t = \frac{2A \times h}{a \times v}$; but (Mechanics, Prop. 248, and Art.

72.) if $s = 16 \frac{1}{2}$ feet, $v = \sqrt{4s \times \frac{1}{2}h} = \sqrt{2sh}$; consequently, $t = \frac{A}{a} \times \frac{2h}{\sqrt{2sh}} = \sqrt{\frac{2}{s}} \times \frac{A}{a} \times \sqrt{h} = ,3526 \times \frac{A}{a} \times \sqrt{h}$.

59. Cor. Hence, the time of emptying any other depth k from the bottom is $,3526 \times \frac{A}{a} \times \sqrt{k}$; consequently, the time of emptying any depth $h - k$ from the top = $,3526 \times \frac{A}{a} \times \sqrt{h - k}$.

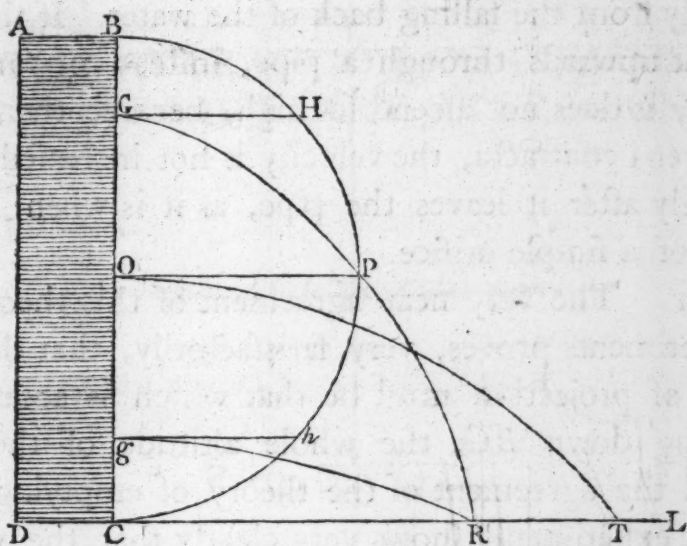
For the time of emptying vessels in general, see the Fluxions, page 217.

PROP. XXXVIII.

To find the distance to which fluids will spout from the side of a vessel placed upon an horizontal plane.

78. Let

78. Let $ABCD$ be a vessel filled with a fluid, and BC be perpendicular to the horizontal plane CL , and upon BC let a semicircle be described, and GH an ordinate perpendicular to BC ; then the distance CR to which the fluid spouts through a very small orifice at G is $= 2GH$. By Art. 72 the velocity at the vena contracta, which is extremely near to the vessel, is that



which a body would acquire in falling down BG ; we are therefore to consider this as the velocity with which the fluid is projected, and not the velocity at the orifice. Now (Mech. Art. 313, and 320) the curve GR described by the fluid is a parabola, and BG is one fourth of the parameter belonging to the point G , which point is the vertex of the parabola, the fluid spouting out horizontally; hence GC is the abscissa and CR it's ordinate, and by the property of the parabola, $4BG \times GC = CR^2$, therefore $CR = 2\sqrt{BG \times GC} = 2GH$, by the property of the circle.

79. Cor. If $Cg = BG$, then $gh = GH$, and the fluid

fluid spouts to the same distance from g as from G . If BC be bisected in O , then the distance CT , to which the fluid spouts, is equal to $2OP = BC$, and this is the greatest distance, OP being the greatest ordinate.

80. If the fluid spout perpendicularly upwards, it ought (Art. 72) to rise to the altitude of the surface of the water in the vessel; but it falls a little short of this, partly from the resistance of the air, and partly from the falling back of the water. If the water spout upwards through a pipe, instead of simply an hole, it does not ascend so high, because there being no vena contracta, the velocity is not increased immediately after it leaves the pipe, as it is when it flows out of a simple orifice.

81. The very near agreement of this theory with experiments proves, very satisfactorily, that the velocity of projection must be that which is acquired in falling down BG , the whole altitude of the fluid. And the agreement of the theory of emptying vessels with experiments shows very clearly that the velocity at the orifice must be that which is acquired in falling through half the altitude of the fluid. Almost immediately, therefore, after the fluid gets out at the orifice, it's velocity is increased in the ratio of $1 : \sqrt{2}$.

SECTION V.

ON THE ATTRACTION OF COHESION, AND ON CAPILLARY ATTRACION.

PROP. XXXIX.

IF two globules of mercury, lying on a smooth plane, be brought to touch, an attraction then takes place, and they immediately rush together and form one complete globule.

82. If the globules be examined with a microscope, no effect is found to take place till they are actually in contact, and then they rush violently together; this therefore can only be accounted for by an attraction which begins at the instant they come into contact.

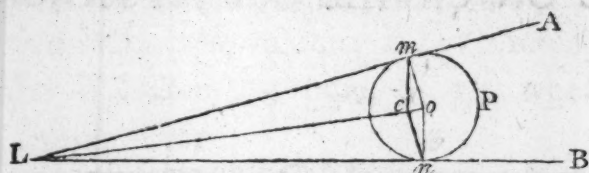
The same effect takes place between two globules of water, when they are laid on a surface on which they can freely move.

PROP.

PROP. XL.

If two glass planes AL, BL, (of which BL is horizontal) be inclined at a very small angle, and just moistened with oil, and a drop P of oil be placed between them, it will move towards their contourse L.

83. This arises from the attraction of cohesion between the drop and the glass planes; for if o be the center of the drop, and om, on represent the attraction



of the drop to each plane, the whole attraction acting perpendicular to the planes, then the compound motion oc , being directed to L will give the drop a motion towards that point. The planes are first moistened, that the drop may move freely.

84. If the point L be elevated, until the motion of the drop ceases, the action oc then becomes equal to the accelerative force of P down the inclined plane LB ; consequently the ratio of the accelerative force upon that inclined plane to the force of gravity, or (Mech. Prop. 61.) the height of the plane to the length, gives the ratio of the attraction oc to the weight of the drop.

PROP. XLI.

If two plane surfaces of metal, &c. be smeared with oil, greafe, &c. and pressed together, they will cohere very strongly.

85. This

85. This effect arises from two causes, the pressure of the surrounding air, in consequence of the air from between being expelled, and from the attraction of cohesion. That it arises partly from the former cause, is manifest from hence, that if two plates be thus put together and cohere pretty strongly in the air, when they be suspended in the receiver of an air pump, after exhausting the air, the under plate will frequently fall; when it does not fall, it shows that there must be an attraction of cohesion, at least equal to the weight of that plate. If the air be expelled by different substances, as oil, turpentine, grease, &c. it is found that the attraction of cohesion is different. This therefore must arise, either from the air being expelled more perfectly by one than the other, or that the attraction is rendered stronger by one than by another.

It is this attraction of cohesion by which the constituent particles of a body, admitting them to be in contact, are kept together. When you break a body, you overcome this attraction, and if you could join the parts together again exactly in the same manner, it would be as strong as before. On this principle we may explain the different degrees of hardness of bodies. Hard bodies may consist of constituent particles which touch in a great part of their surfaces, and thus their attraction may be very great. The constituent particles of soft bodies may touch in a few points, and thus their attraction will be weakened. Solids are supposed to be dissolved in menstrua, from the attraction of cohesion between the particles of the fluid and body being greater than the attraction between the constituent particles of the solid.

PROP.

PROP. XLII.

There is an attraction of cohesion between water and glass.

86. For take a piece of very clean glass, and hold it in an horizontal position, and a drop of water will remain suspended from its under side.

PROP. XLIII.

The constituent particles of water attract each other.

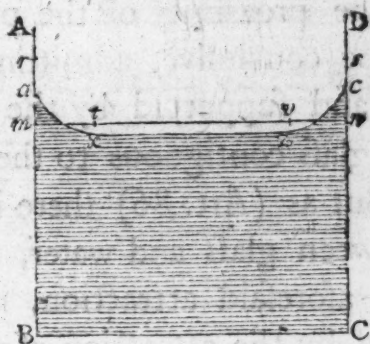
87. For a small quantity of water always forms itself into a globule, if no external circumstance tend to prevent it; this can arise only from the mutual attraction of it's parts. If the quantity be large, the upper surface will grow flat, from it's gravity overcoming the attraction of it's parts, but the sides will still be convex.

PROP. XLIV.

There is an attraction between water and glass, at a distance.

88. Pour water into the glass vessel *ABCD*, and if no attraction took place between the water and glass, the surface *mn* would be horizontal. Take *mr = mt = ns = nv*, and suppose the glass to act upon the water through these distances; then the glass *mr* will attract the surface *mt*, and *ns* will attract *nv*, and part of this attraction acting upwards, the gravity
of

of the columns of water under mt , vn will be diminished by the attraction, and more diminished the



nearer to the sides, consequently their length must be increased in order to be in equilibrium with the other columns whose gravity is not diminished; hence the water will rise in a curve ax , cz from the points x and z as far as the attraction extends, and the other part xz will be horizontal; now when water is put into a glass vessel, the surface of the fluid puts on the form axz ; we conclude, therefore, that glass acts upon water at a distance. In like manner, if any piece of glass be immersed in water, the water will rise on each side of the glass.

89. Cor. 1. Hence, if two pieces of glass, parallel to each other, be immersed in the vessel, the water will rise against each. Let them be so near, that the two curves ax , cz may just meet, then will a certain quantity of the fluid between them be raised above the surface of the fluid in the vessel. Bring them nearer, and as the glass still exerts the same attraction, it must, upon this principle, raise the same quantity of water, and therefore the altitude will be inversely

as

as the distance of the planes; for the same attraction being exerted, there must be the same quantity of fluid supported, consequently the altitude of the fluid will increase as the proximity of the planes decrease. This seems to be conclusive, admitting the water to be both raised and supported by the attraction of a small part of the glass contiguous to the upper surface of the fluid. But as (Art. 86) there is an attraction of cohesion between glass and water, after the water is raised by the aforesaid attraction, it will then, in part be supported by the attraction of cohesion. An additional quantity of water may therefore probably be *supported* from this cause.

90. Cor. 2. If the glass planes be *inclined* to each other, then it follows, from these principles, that as the distance between the glasses decreases, the altitude of the water will increase. Mr. HAUKEBEE informs us, that he very accurately measured the abscissas and ordinates of the curve formed by the upper surface of the water between the glass planes, and concluded it to be the common hyperbola, having the surface of the fluid, and concurrence of the planes, for it's asymptotes. Now if we admit that the water is raised, and also supported, by the attraction of the glass lying just above the surface of the water, the curve ought to be the common hyperbola; for if we divide the water into laminæ of the same thickness, then there being the same attraction exerted upon each, the same quantity will be supported, and therefore the altitudes (or ordinates) must be inversely as the lengths of the laminæ, or distances (abscissas) of the laminæ from the concurrence of the planes, which is the

the property of the hyperbola between the above mentioned asymptotes.

PROP. XLV.

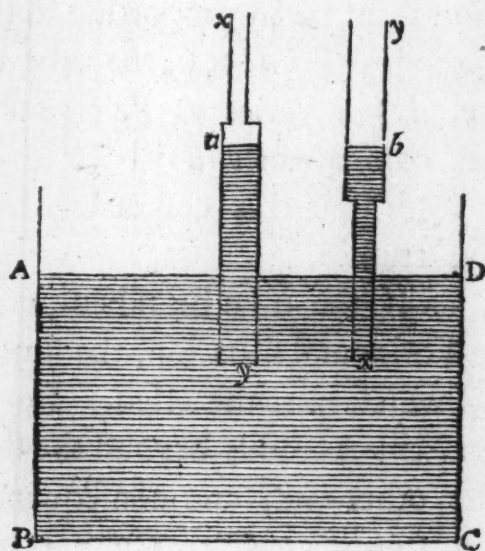
If very small glass tubes, called capillary tubes, be dipped in water, the water is found to stand in them, above the level of that in the vessel, at altitudes which are either accurately or nearly in the inverse ratio of their diameters.

91. If we admit the fluid to be raised and supported only by an annular surface of the glass contiguous to the upper surface of the water, the ratio ought to be accurately so. For let d = the diameter of the tube, a = the altitude of the water in it; then the breadth of the attracting annulus being constant (it being the distance to which the attraction of the glass reaches), the area of this annulus, or the attracting surface, will be in proportion to the circumference of the tube, and consequently to the diameter d ; also the quantity of attraction must be in proportion to the quantity of water supported, or to $d^2 \times a$, the tube being cylindrical; hence, $d^2 \times a$ varies as d , therefore $d \times a$ is constant, and consequently a varies inversely as d . Experiments show that this conclusion is accurately, or very nearly, true.

92. Cor. 1. If the tube be taken out of the fluid and laid in an horizontal situation, the fluid will recede from that end which was immersed. For at that end there is no attracting annulus beyond the fluid, whereas there is at the other end, and consequently the fluid will be drawn towards the empty part of the tube,

until the length of the other end, left free from the fluid, be equal to the distance to which the attraction reaches.

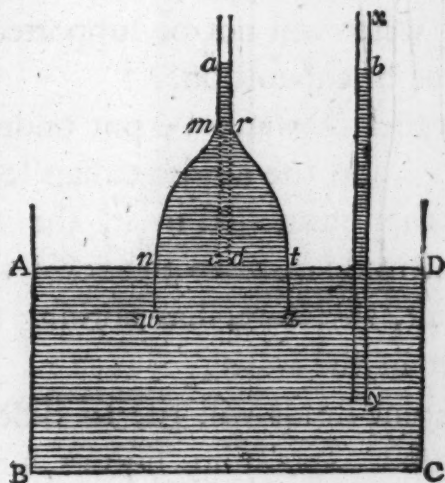
93. Dr. HAMILTON, in his *Essays*, thinks, that the fluid is not supported by the attraction of the annular surface of the tube contiguous to the upper surface of the water, but by the annulus at the bottom, contiguous to the bottom of the tube; this he supposes will first draw up a plate of water immediately under it, and then a succession of plates, till the weight of the whole is equivalent to the attraction of that annulus. If this were the case, the quantity supported, and consequently the altitude of the fluid, would depend upon the orifice at the bottom, whereas, experiments show that the altitude at which the fluid is supported depends upon the diameter of the tube at the upper surface of the fluid, without any regard to the form of the tube below it. See other objections to this solution in Mr. PARKINSON'S *Hydrostatics*, page 40.



94. Dip the tube xy (being two cylindrical tubes joined

joined together) into water, and suppose the fluid to rise to a by the capillary attraction; invert the tube, and the fluid will rise to b , the same altitude where the diameter is the same. If we admit the principle in Art. 91, the annular surface being the same in both, the same quantity of fluid ought to be supported, whereas, when the small end is downwards, the quantity supported is less. But if we admit the fluid to be partly retained by the attraction of cohesion, the quantity of the surface being less with the smaller end downwards, the whole support is less, and therefore the quantity supported ought to be less. This seems to be in favour of the support being partly by the attraction of cohesion.

95. If the water rise in the capillary tube xy to b , and another vessel az be put into the water, having



the upper part capillary, and equal to that of xy , but the lower part of any size; then if the air be drawn out

of this vessel by suction until the water enters into the capillary part, it will stand at the same altitude as in the tube xy , after the suction ceases and the air is admitted into the capillary part. Here the cylindrical part ad is supported by the same power as the water in the other tube, and the other part myn , rtd is supported by the pressure of the air upon the surface of the water in the vessel, in consequence of the air being drawn from within; and this is proved from hence, that if the whole be put under a receiver of an air pump, and the air be exhausted from the surface of the vessel, the water will not be supported. Dr. JURIN says, that the water will be supported in vacuo; but in his time, the air pumps would not exhaust sufficiently to determine this point, for the altitude to which the water rises being very small, it requires a considerable degree of exhaustion before the water will fall. The pumps which are now made, show that the water will not be supported after a considerable degree of exhaustion.

96. If a vessel of water be put under the receiver of an air pump, and the air be exhausted, and capillary tubes be then immersed in it, the water will rise to the same altitude as when the vessel was exposed to the air; the air, therefore, has nothing to do in causing the ascent of the water.

97. Different fluids will rise to different altitudes in the same tube. Spirituous liquors, which are lighter than water, rise to a less height than water, which, of all fluids, appears to rise to the greatest height. This can arise only from the different degrees of attraction of these fluids to glass.

98. The

98. The diameter of the tube multiplied into the altitude of the fluid in it is (Art. 91) accurately or very nearly a constant quantity, which, by experiment, is found to be ,053 of an inch.

99. The diameter of a capillary tube is thus found to a very great degree of accuracy. Put into the tube some mercury whose weight in grains is w , and let it occupy a length of the tube l ; then if 13,6 be the specific gravity of mercury (which it is when purest)

that of water being 1, the diameter $= \sqrt{\frac{w}{l}} \times ,01923$.

For if d = the diameter, the content of the mercury $= d^2 l \times ,7854$, and as one cubic inch of mercury weighs 3443 grains, we have $1 : 3443 :: d^2 l \times ,7854$

$: w$, wence $d = \sqrt{\frac{w}{l}} \times ,09123$. This rule is given

by Mr. ARWOOD in his *Analysis*.

PROP. XLVI.

There is a small attraction of cohesion between mercury and glass.

100. For a very small globule of mercury will adhere to the under side of a clean piece of glass.

PROP. XLVII.

There is a strong attraction of the constituent particles of mercury towards each other.

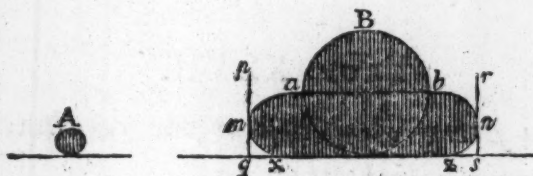
101. For a small quantity of mercury laid upon a piece of glass will, as to sense, form itself into a perfect

fect sphere. If two of these spheres be brought into contact, at the instant they touch they will rush together with a surprising quickness, and form a single sphere. This can only be explained upon the principle of the mutual attraction of all the parts. As you increase the quantity of mercury, it will begin to deviate from a perfect sphere, and grow flatter on the upper side, arising from the gravity of the mercury becoming sensibly greater than the mutual attraction of it's constituent particles; and when the quantity becomes considerable, the upper surface will not sensibly differ from a perfect plane, but the sides will retain their convexity.

PROP. XLVIII.

If mercury be put into a glass vessel, it will stand lowest at the sides, and rise in a curve till the surface becomes, as to sense, a plane.

102. This arises from the mercury attracting itself by a greater force than it is attracted to the glass, and may be thus explained. A very small quantity *A* of mer-



cury laid upon glass, will, as to sense, form itself into a perfect sphere. If we take a large quantity *B*, it will
not

not preserve it's spherical form, the force of gravity destroying that figure by overcoming the mutual attraction of the particles, and the mercury will put on the form $xmabnz$; and if two pieces of glass pq , rs be made to touch the mercury at m and n , the form will not be sensibly altered. If therefore we take a glass vessel $pqsr$ and put mercury into it, the upper surface will still be in the form $mabn$, for it can manifestly make no difference whether we put the glass to the mercury, or the mercury to the glass; except that in the latter case, the spaces mqx , nsr will be filled with mercury, which can have no effect upon the upper surface.

103. Cor. 1. Hence, if a piece of glass be dipped into mercury, the mercury will be depressed on each side of the glass, in the same manner.

104. Cor. 2. If the two pieces of glass, pq , rs be brought so near, that the depressed parts of the mercury may meet, the mercury will be depressed to a distance which is inversely as the distance of the glasses, accurately or nearly so.

105. If small capillary tubes be put into a vessel of mercury, the fluid in the tubes will be depressed to distances below the surface of the fluid in the vessel, which are found, by mensuration, to be inversely as their diameters, accurately or nearly so.

106. If two glass planes, inclined at a small angle, be put into a vessel of mercury, the mercury between them will be depressed below the surface of the mercury without the planes, and that depression is found (as nearly as it can be determined by mensuration) to be inversely as the distance from the concurrence of the

planes, and therefore the curve is an hyperbola, having the concourse of the planes for one asymptote, and the surface of the mercury against the planes without, for the other asymptote.



SECTION

SECTION VI.

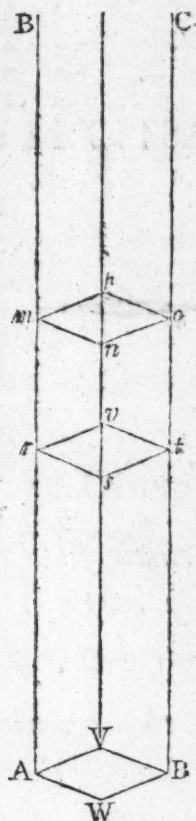
ON ELASTIC FLUIDS.

PROP. XLIX.

IF the particles of an elastic fluid repel each other with forces varying inversely as the n^{th} power of their distances, and d represent the density of any part, and c the compressive force upon it, then c varies as $d^{\frac{n+2}{3}}$.

107. Let $ABCD$ be a square column of the fluid, $mnop$, $rstv$ two sections parallel to the base (and consequently each a square equal to the base) whose distance mr is equal to mn , one side of the square, then will the fluid contained between these sections be a cube. Let d be the density of the fluid in this cube, supposed to be indefinitely small, c the compressive force on it, and r the distance of the particles; then one side of the cube being indefinitely small, d , c and r may be considered as the same for the whole of this cube. Now the number of particles
in

in mn is as $\frac{1}{r}$, consequently the number in the whole section $mno p$ is as $\frac{1}{r^2}$. Also the repulsive force of all



the particles in $mno p$, being as the number of particles and force of each conjointly, is as $\frac{1}{r^2} \times \frac{1}{r^n} = \frac{1}{r^{n+2}}$; and as the repulsive force of each particle acts in every direction, this repulsive force acting upwards must be equal to the compressive force which it sustains, or c will vary as $\frac{1}{r^{n+2}}$; they will not necessarily be equal, because

because $\frac{1}{r^{n+2}}$ does not represent the *quantity* of the repulsive force, only what it is proportional to. Also the number of particles in the cube is as $\frac{1}{r^3}$, and therefore (Art. 4) d varies as $\frac{1}{r^3}$, hence $d^{\frac{1}{3}}$ varies as $\frac{1}{r}$, and $d^{\frac{n+2}{3}}$ varies as $\frac{1}{r^{n+2}}$; but a varies as $\frac{1}{r^{n+2}}$; consequently c varies as $d^{\frac{n+2}{3}}$.

108. It appears by experiment, that the compressive force of the atmospheric air varies as the density, or c varies as d , therefore in this case $\frac{n+2}{3} = 1$, and hence $n = 1$, consequently the particles of air repel each other with forces which vary inversely as their distances. Also, as the compressive force of air is equal to it's elastic force, these balancing each the other, the elastic force must vary as the density.

109. It is manifest that there can be no fluid whose density varies in any inverse ratio of the compressive force, that is, you can never, by increasing the compressive force, diminish the density, as any increase of the compressive force must compress the fluid into a less space, and therefore increase the density, unless the particles of the fluid were absolutely in contact, in which case the density would remain the same under any pressure, which is probably not the case with any fluid. Hence $n+2$ must be always positive, that is, n must be some whole positive number, or a negative number less than 2, in order to constitute a fluid consisting of particles which repel each other. If we admit
water

water to be compressible in a very small degree, the particles must be kept at a distance by some repulsive force, and n must be very nearly $= -2$; hence, upon this supposition, the repulsive force of the particles of water varies nearly as the square of their distances.

PROP. L.

Air has weight.

110. If a vessel be exhausted of air, and balanced at one end of a beam, upon admitting the air the vessel preponderates. This clearly proves that air has weight, and therefore it must press upon all bodies; and from the weight necessary to balance the vessel after the air is admitted, compared with the weight of the vessel of water, we get the specific gravity of air to that of water (Art. 3), which is, as 1 to about 885 in the mean state of the air, or when the barometer stands at $29\frac{1}{2}$ inches, according to Mr. HAUKEBEE. Others have made the specific gravities as 1 to 850, when the barometer stands at 30 inches.

LEMMA.

111. If $a : b :: b : c :: c : d :: \&c.$ then by EUCLID, B. 5. p. 12. $b : c :: b + c + d + \&c. : c + d + e + \&c.$ hence, $a : b :: b + c + d + \&c. : c + d + e + \&c.$; for the same reason, $b : c :: c + d + e + \&c. : d + e + f + \&c.$ and so on to the end of the series. Hence, vice versâ, if $a : b :: b + c + d + \&c. : c + d + e + \&c.$ and $b : c :: c + d + e + \&c. : d + e + f + \&c.$ and so on, then will $a : b :: b : c :: c : d :: \&c.$

PROP.

PROP. LI.

If the force of gravity be considered as constant, and altitudes from the Earth's surface be taken in arithmetic progression, the corresponding densities of the air will decrease in geometric progression.

112. Conceive the whole atmosphere to be divided into an indefinite number of laminæ of equal thickness, parallel to the Earth's surface; and let a , b , c , &c. represent the respective densities of these laminæ, beginning at the surface of the earth; then the compressive force on the laminæ a , b , c , &c. will be proportional to the weight incumbent upon each, that is, the sum of the weights of all the laminæ above; but the weight of each lamina is as the density $\times$ it's thickness, or, as the thickness is the same, as it's density; hence the compressive forces on a , b , c , &c. will be as the sum of all the quantities which represent the densities above them, or as $b+c+d+\&c.$ $c+d+e+\&c.$ $d+e+f+\&c.$ &c. &c. But (Prop. LVII) the compressive force of the air is as it's density; hence, $a : b :: b+c+d+\&c. : c+d+e+\&c.$ and $b : c :: c+d+e+\&c. : d+e+f+\&c.$ and so on; hence, by the above Lemma, $a : b :: b : c :: c : d :: d : e :: \&c.$ Now the laminæ being of the same thickness, the last proportion shows that as you ascend by equal spaces, or in arithmetical progression, the densities decrease in geometrical progression.

PROP. LII.

Given the density of the air, to find the corresponding altitude; and the converse.

113. By

113. By the nature of logarithms, if the natural numbers be in geometrical progression, their logarithms are in arithmetical progression; hence, as the altitudes increase in arithmetical progression whilst the corresponding densities of the air decrease in geometrical progression (Art. 112), it follows, that the altitudes increase as the logarithms of the densities decrease. Hence, if at the altitudes x and y the densities be m and n times less, or be $\frac{1}{m}$ and $\frac{1}{n}$, the density

of the surface being unity, we have $x : y :: \log. \frac{1}{m} :$

$:\log. \frac{1}{n} :: \log. m : \log. n$. Now Mr. COTES (*Hyd.*

p. 103) collected from experiment, that at the altitude of 7 miles, the density is 4 times less than the density at the surface; hence, if $y = 7$, $n = 4$, we have, $x : 7 :: \log.$

$m : \log. 4$, therefore $x = 7 \times \frac{\log. m}{\log. 4} = 11,626 \times \log. m$;

if therefore the density $\frac{1}{m}$ be given, we know x . Also

$\log. m = \frac{x}{7} \times \log. 4$, therefore (Flux. Art. 109.) $m =$

$4^{\frac{x}{7}}$, consequently if the altitude be given, the density

$\frac{1}{m}$ will be known. But the density is inversely as

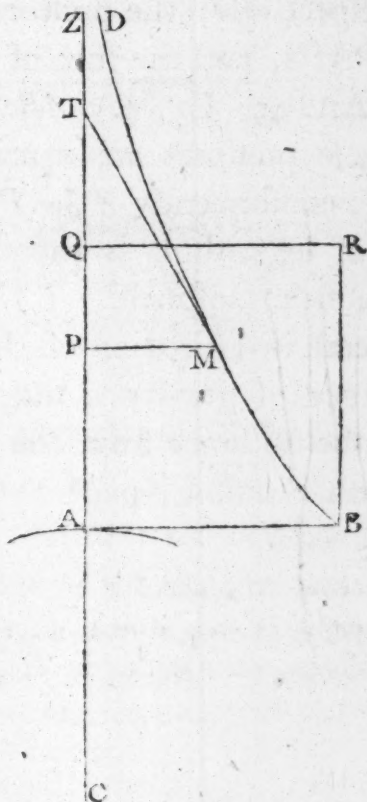
the rarity, that is, if the density be 4 times less, or be expressed by $\frac{1}{4}$, the rarity will be 4 times greater, or will be expressed by 4; hence m may express the rarity of the air, that at the surface being unity. This rule is not accurate, because it supposes the compressive force

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force of the air to be as it's density, which is not true, unless the temperature be the same.

If it should appear that the altitude at which the density is 4 times less than at the surface, be not 7 miles, then 11,626 must be altered in the ratio of 7 to that altitude.

114. Let CA be the radius of the Earth, which produce to Z , draw AB perpendicular to CA and let it represent the density of the air at the surface, and PM represent the density at any altitude AP , and let



BMD be a curve passing through the extremities of all the ordinates PM . Then (Art. 112) as AP increases in arithmetic progression, the density PM will decrease in geometric progression; hence PM is the
number

number corresponding to the logarithm AP , and BD is called the logarithmic curve, whose subtangent PT is the modulus of the system. Let AQ be the altitude of an homogeneous* atmosphere whose density is AB , and complete the parallelogram $BAQR$. Now consider the whole atmosphere AZ , and the homogeneous atmosphere AQ to be divided into an indefinite number of laminæ of equal thickness; then (Art. 112) the whole pressure on AB in both cases may be measured by the sum of all the densities of these laminæ, and the density being as the lamina PM and AB respectively, the pressures will be as the sum of all the PM 's, and the sum of all the AB 's, or as (Fluxions, Art. 49, Ex. 4.) $AB \times PT$, and $AB \times AQ$; but these pressures are equal; hence $AB \times PT = AB \times AQ$, consequently $AQ = PT$; the modulus of this system of logarithms is therefore the altitude of an homogeneous atmosphere.

For the general investigation of the density of the air, when the force of gravity is supposed to vary as any power of the distance from the Earth's center, see the treatise on Fluxions, page 216.

* An homogeneous atmosphere is an atmosphere supposed to be of the same weight as that which surrounds the Earth, and whose density is uniform and equal to the density of the air at the Earth's surface.



SECTION

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SECTION VII.

ON THE BAROMETER.

PROP. LIII.

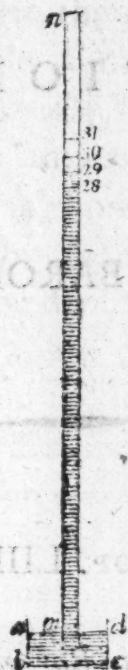
To make a Barometer.

115. If a glass tube above 31 inches long, hermetically sealed at one end, be filled with mercury, and then it's open end be immerfed in a bason of the same fluid, the altitude at which the mercury will stand in the tube above the surface of the mercury in the bason is between 28 and 31 inches. A tube thus filled is called a *Barometer*.

PROP. LIV.

The mercury is suspended in the tube of a barometer by the pressure of the air upon the surface of the mercury in the bason.

116. For if a barometer *mn* be put under the receiver of an air pump, and the air be exhausted, as you



continue to exhaust the air, and consequently to diminish it's pressure upon the surface of the mercury in the basin, the mercury in the tube will continue to descend, and when no sensible quantity of air is left, the altitude of the mercury will not be sensibly above that in the basin; and upon admitting the air again into the receiver, the mercury will rise in the tube to it's former height.

As the density of the air, and consequently it's compressing force, is subject to a variation, the altitude of the mercury must be subject also to a corresponding variation; it is always however contained between

between the limits of 28 and 31 inches. A tube thus filled is therefore graduated from 28 to 31 inches.

117. GALILEO was the first person who discovered the pressure of the air. He found by experiment, that water could be raised, by the common pump, to a certain height, and no higher; whereas, had nature abhorred a vacuum, according to the opinion of some of the philosophers at that time, the water might have been raised to any height. He conjectured therefore, that it was owing to the air's gravitation; the truth of which was afterwards confirmed by his pupil TORRICELLIUS, who considered, that if the pressure of the air could support a column of water 35 feet high, which is about the mean height to which a pump can raise water, it could suspend a column of mercury, whose density is about 14 times as great, only about one 14th part of 35 feet high, or about 30 inches; he accordingly tried the experiment, and found that the mercury stood at the altitude which he expected. Thus he clearly proved the gravitation of the air; and hence this is called the *Torricellian* experiment; and the vacuum which is left above, when the mercury descends from the top of the tube after immersing it in the basin, is called the *Torricellian* vacuum. When the tube is filled with great care, this vacuum is supposed to be the most perfect that can be made.

PROP. LV.

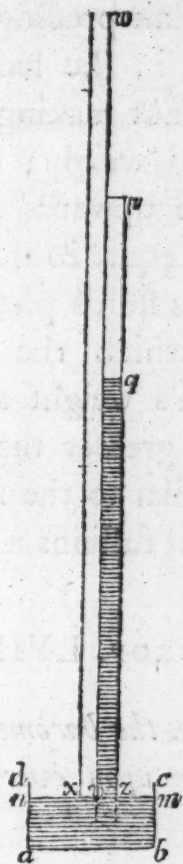
To find the height of an homogeneous atmosphere.

118. The mercury in the tube of a barometer is (Art. 116) sustained by the pressure of the air; and (Art. 31) when two fluids communicate, the altitudes at which they stand are inversely as their specific gravities. Let us take the specific gravity of air to that of water as 1 : 850, the barometer standing at 30 inches. Now if we take the specific gravity of water to mercury as 1 : 14, we shall have the specific gravity of air to that of mercury as 1 : 12390; hence, (Art. 31) $1 : 12390 :: 30 \text{ in.} : 12390 \times 30 = 371700 \text{ in.} = 5,63 \text{ miles}$, the height of an homogeneous atmosphere, or an atmosphere of the same weight as the present atmosphere, and whose specific gravity is every where the same as that of the air at the earth's surface. If we take the specific gravities of air and water as 1 : 885, when the barometer stands at $29\frac{1}{2}$ inches, we shall have the altitude of an homogeneous atmosphere 5,77 miles. The specific gravity of mercury has been here supposed 14, that of water being 1; but when the mercury is very pure, its specific gravity has been found to be only 13,6. To determine with accuracy the height of an homogeneous atmosphere by this method, the specific gravity of the mercury in the barometer, at the time of observation, should be determined, as it is subject to a small variation from the different temperatures of the air.

PROP.

PROP. LVI.

The weight of the mercury in the barometer (the tube being cylindrical) above the level of that in the basin, is equal to the weight of a cylinder of air of the same base reaching to the top of the atmosphere.



119. Let qz be the altitude of the mercury in the tube pv ; take a cylindrical column xvw of the air, whose base xv is equal to vz that of the mercury. Now the section $xxvm$ of mercury being at rest, every point thereof must be equally pressed, and therefore equal parts must be equally pressed; but the pressures

on xv , vz arise from the incumbent columns xw of air and vg of mercury, and these being perpendicular cylindrical columns the pressures are equal to their weights; consequently the weights of these columns are equal.

Some have found it difficult to conceive why the weight of the mercury in the tube should not be equal to the weight of the air pressing upon the *whole* surface of the mercury in the basin. This difficulty has arisen from their not making a proper distinction between pressure and weight; the column of mercury gives a pressure upwards to the surface of the mercury in the basin equal to the weight of the whole incumbent air, but as fluids press equally in all directions, this pressure which the mercury gives is as much greater than its weight as the surface of mercury in the basin is greater than the orifice of the tube. It is a fact similar to the hydrostatical paradox, where a smaller weight sustains a greater.

PROP. LVII.

When the mercury in the barometer stands at 30 inches, the pressure of the air upon every square inch is about 15 lb. avoirdupoise.

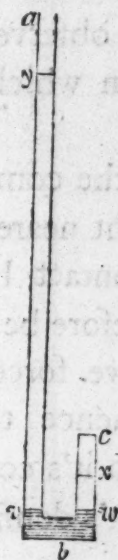
120. For by the last Article, a column of air, of mean density, whose base is 1 square inch, presses as much as a column of mercury of the same base 30 inches high, the weight of which is about 15 lb. avoirdupoise.

121. Cor. Hence, if we take the surface of a middle size man to be $14\frac{1}{2}$ square feet, when the air is lightest
it's

it's pressure on him is 13,2 tons ; and when heaviest, it is 14,3, the difference of which is 2464 lb. This difference of pressures must greatly affect us in regard to our animal functions, and consequently in respect to our health, more especially when the change takes place in a short time. The pressure of the air upon the whole surface of the Earth is about 77670297973563429 tons.

PROR. LVIII.

The density of the air is in proportion to the force which compresses it.



122. Let abc be a glass cylindrical tube, hermetically sealed at c , and let the bottom be covered with mercury, whilst the air in wc is in its natural state. Pour in mercury at a and it will force the mercury to rise in wc , and continue to pour in, till the mercury stands at y as high above the point

F 4

 x to

x to which it has now risen in wc , as the altitude of the mercury in the common barometer; then that column of mercury (Art. 119) is equivalent to the weight of the column of air incumbent upon it, hence the pressure against the air in cx is now twice as much as it was against the air in cw , and cx is observed to be $= \frac{1}{2}cw$; hence the air being compressed into half the space, the density is doubled. In like manner, if another column of mercury of the same altitude be added, cx is found to be $= \frac{1}{3}cw$; thus the compressing force is made three times greater, and the density is three times greater. In this manner, the compressing force is found in any other case to be in proportion to the density. The same is observed to be true in all kinds of factitious airs, upon which experiments have been made.

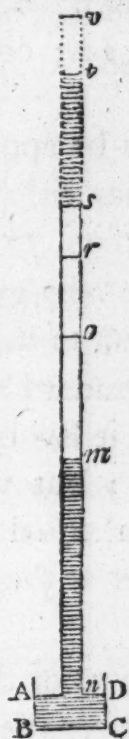
123. By increasing the compressing force of the air, the particles are brought nearer together, but are kept from coming into contact by their repulsive force; these forces must therefore be equal, when the fluid is at rest. The repulsive force is what we usually call the air's elasticity; hence the elasticity of the air being in proportion to its compressive force, must be also in proportion to its density.

PROP. LIX.

If the tube ns of a barometer be perfectly cylindrical, and be in part only filled with mercury, and then its open end be immersed in a basin of the same fluid, the mercury will sink below the point, called the standard altitude, or the point at which it would have stood if no air had been

been left in; and the standard altitude will be to the depression below that altitude as the space occupied by the air after the immersion to the space occupied before.

124. Let rs be equal to the space occupied by the air before the tube is immersed, or when the air is in it's natural state; after the immersion of the tube into the basin $ABCD$ let the mercury sink to m ; then the



air which, in it's natural state, occupied the space rs , now occupies the space ms , and the space occupied by the same quantity being inversely as the density, or (Art. 123) inversely as the elasticity, we have the elasticity in rs : the elasticity in sm :: sm : rs . Let no be the height at which the mercury would

would have stood if no air had been left in the tube, or the height of the mercury in the barometer. Then (Art. 122) the compressive force of the air, and (Art. 123) consequently it's elasticity, when it occupied the space rs , would support a column of mercury no , because the air, when it occupied that space, was in it's natural state; and the elasticity of the air when it occupies the space sm would support a column of mercury mo , because it depresses the mercury from o to m ; hence the elasticity of the air in rs : the elasticity in $sm :: on : om$; consequently $on : om :: sm : sr$.

This proposition may be applied to the solution of two problems; for we may either give the quantity of air left in before immersion, to find the altitude of the mercury after; or we may give the altitude of the mercury after immersion, to find the quantity of air left in before. As the standard altitude no (Art. 116) is subject to a variation, it has been usual in this case to assume it 30 inches; but when accuracy is required, it must be taken equal to the height of the mercury in the barometer at the time.

Ex. 1. Let the length ns of the tube be 35 inches, and the depression om below the standard altitude no ($=30$ in.) be 10 inches, to find the quantity of air left in before inversion.

As $ns=35$, and $no=30$, we have $so=5$; also $om=10$; hence $sm=15$; therefore, $30 : 10 :: 15 : rs$
 $= \frac{10 \times 15}{30} = 5$ inches.

Ex. 2.

Ex. 2. Let 5 inches of air be left in the same tube before inversion, to find the altitude of the mercury after.

In this case the point m being unknown, the second and third terms of the proportion are unknown; put therefore $x = om$, then $x + 5 = sm$; hence $30 : x :: x + 5 : 5$, therefore $x^2 + 5x = 150$, consequently $x = 10$ or -15 . The answer $+10$ shows that the mercury will stand at 10 inches below o ; and the answer -15 shows, that if the tube were continued to v , and ot taken equal to 15 inches, and the space st were filled with mercury, the space tv above being a vacuum, that this column st of mercury would also be supported by the elasticity of the air in sm . In fact, $st = om$, and therefore the elasticity of the air which depresses a column om must necessarily sustain an equal column st .

The experiments agreeing with the conclusions here deduced, it follows that the compressive force of the air is as it's density, that being the principle upon which the demonstration is founded.

PROP. LX.

If a be the altitude of the mercury in a barometer at the bottom of an hill, and b the altitude at the top, the altitude of the hill will be $11,626 \times \log. \frac{a}{b}$ miles.

125. For the weight of mercury suspended by the pressure of the air at the bottom : the weight suspended at the top :: $a : b :: 1 : \frac{b}{a}$; hence the compressive
forces

forces of the air at these two points, by sustaining these columns of mercury, must be as $1 : \frac{b}{a}$; but the compressive forces are as (Art. 122) the densities; therefore the densities below and above are as $1 : \frac{b}{a}$, consequently the density above will be $\frac{b}{a}$, that at the surface being unity. Hence the rarity will be $\frac{a}{b}$; substitute this for m (Art. 113.) and the altitude

$x = 11,626 \times \log. \frac{a}{b}$. The difference of the temperatures of the air at the bottom and top is not here considered, which will make a small alteration. This correction may be applied, by observing with two thermometers the temperatures at each point, and then allowing for that difference; but the investigation of this falls not within our present plan.



Dr. HALLEY's *Account of the Rising and Falling of the Mercury in a Barometer, upon the Change of Weather.*

126. To account for the different heights of the mercury at several times, it will not be unnecessary to enumerate some of the principal observations made upon the barometer.

1st. The first is, that in calm weather, when the air is inclined to rain, the mercury is commonly low.

2ly. That in serene, good, settled weather, the mercury is generally high.

3ly. That upon very great winds, though they be not accompanied with rain, the mercury sinks lowest of all, with relation to the point of the compass the wind blows upon.

4ly. That *ceteris paribus*, the greatest heights of the mercury are found upon easterly and north-easterly winds.

5ly. That in calm frosty weather, the mercury generally stands high.

6ly. That after very great storms of wind, when the quicksilver has been low, it generally rises again very fast.

7ly. That the more northerly places have greater alterations of the Barometer than the more southerly.

8ly. That within the tropicks and near them, those accounts we have had from others, and my own observations at St. Helena, make very little or no variation of the height of the mercury in all weathers.

Hence

Hence I conceive, that the principal cause of the rise and fall of the mercury, is from the variable winds which are found in the temperate zones, and whose great inconstancy here in England is most notorious.

A second cause is the uncertain exhalation and precipitation of the vapours lodging in the air, whereby it comes to be at one time much more crouded than at another, and consequently heavier; but this latter in a great measure depends upon the former. Now from these principles I shall endeavour to explicate the several phænomena of the barometer, taking them in the same order I laid them down.

1st The mercury's being low inclines it to rain, because the air being light, the vapours are no longer supported thereby, being become specifically heavier than the medium wherein they floated; so that they descend towards the Earth, and in their fall meeting with other aqueous particles, they incorporate together and form little drops of rain. But the mercury's being at one time lower than at another, is the effect of two contrary winds blowing from the place where the barometer stands; whereby the air of that place is carried both ways from it, and consequently the incumbent cylinder of air is diminished, and accordingly the mercury sinks. As for instance, if in the German ocean it should blow a gale of westerly wind, and at the same time an easterly wind in the Irish sea, or if in France it should blow a northerly wind, and in Scotland a southerly, it must be granted me that, that part of the atmosphere impendent over England would thereby be exhausted and attenuated, and the mercury would subside, and the vapours which before floated

floated in those parts of the air of equal gravity with themselves, would sink to the Earth.

2ly. The greater height of the barometer is occasioned by two contrary winds blowing towards the place of observation, whereby the air of other places is brought thither and accumulated; so that the incumbent cylinder of air being increased both in height and weight, the mercury pressed thereby must needs rise and stand high, as long as the winds continue so to blow; and then the air being specifically heavier, the vapours are better kept suspended, so that they have no inclination to precipitate and fall down in drops; which is the reason of the serene good weather, which attends the greater heights of the mercury.

3ly. The mercury sinks the lowest of all by the very rapid motion of the air in storms of wind. For the tract or region of the Earth's surface, wherein these winds rage, not extending all round the globe, that stagnant air which is left behind, as likewise that on the sides, cannot come in so fast as to supply the evacuation made by so swift a current; so that the air must necessarily be attenuated when and where the said winds continue to blow, and that more or less according to their violence; add to which, that the horizontal motion of the air being so quick as it is, may in all probability take off some part of the perpendicular pressure thereof: and the great agitation of its particles is the reason why the vapours are dissipated, and do not condense into drops so as to form rain, otherwise the natural consequence of the air's rarefaction.

4ly. The mercury stands the highest upon an easterly or north-easterly wind, because in the great Atlantick ocean,

ocean, on this side the 35th degree of north latitude, the westerly and south-westerly winds blow almost always Trade, so that whenever here the wind comes up at east and north-east, it is sure to be checked by a contrary gale as soon as it reaches the ocean; wherefore according to what is made out in our second remark, the air must needs be heaped over this island, and consequently the mercury must stand high, as often as these winds blow. This holds true in this country, but is not a general rule for others where the winds are under different circumstances; and I have sometimes seen the mercury here as low as 29 inches upon an easterly wind, but then it blew exceeding hard, and so comes to be accounted for by what was observed upon the third remark.

5ly. In calm frosty weather the mercury generally stands high, because (as I conceive) it seldom freezes but when the winds come out of the northern and north-eastern quarters, or at least unless those winds blow at no great distance off; for the northern parts of Germany, Denmark, Sweden, Norway, and all that tract from whence north-eastern winds come, are subject to almost continual frost all the winter; and thereby the lower air is very much condensed, and in that state is brought hitherwards by those winds, and being accumulated by the opposition of the westerly wind blowing in the ocean, the mercury must needs be pressed to a more than ordinary height; and as a concurring cause, the shrinking of the lower parts of the air into lesser room by cold, must needs cause a descent of the upper parts of the atmosphere to reduce the cavity made by this contraction to an *æquilibrium*.

6ly. After

6ly. After great storms of wind, when the mercury has been very low, it generally rises again very fast. I once observed it to rise $1\frac{1}{2}$ inch in less than 6 hours after a long continued storm of south-west wind. The reason is, because the air being very much rarefied, by the great evacuations which such continued storms make thereof, the neighbouring air runs in the more swiftly to bring it to an *æquilibrium*; as we see water runs the faster for having a great declivity.

7ly. The variations are greater in the more northerly places, as at Stockholm greater than at Paris (compared by Mr. PASCAL *) because the more northerly parts have usually greater storms of wind than the more southerly, whereby the mercury should sink lower in that extreme; and then the northerly winds bringing the condensed and ponderous air from the neighbourhood of the pole, and that again being checked by a southerly wind at no great distance, and so heaped, must of necessity make the mercury in such case stand higher in the other extreme.

8ly. Lastly, this remark, that there is little or no variation near the equinoctial, as at Barbadoes and St. Helena, does above all others confirm the hypothesis of the variable winds being the cause of these variations of the height of the mercury; for in the places above named there is always an easy gale of wind blowing nearly upon the same point, viz. E.N.E. at Barbadoes, and E.S.E. at St. Helena, so that there being no contrary currents of the air to exhaust or accumulate it, the atmosphere continues much in the same state: however upon hurricanes (the most violent

* Equilibre des Liqueurs.

violent of storms) the mercury has been observed very low, but this is but once in two or three years, and it soon recovers its settled state of about $29\frac{1}{2}$ inches.

The principal objection against this doctrine is that I suppose the air sometimes to move from those parts where it is already evacuated below the *æquilibrium*, and sometimes again towards those parts where it is condensed and crouded above the mean state, which may be thought contradictory to the laws of statics, and the rules of the *æquilibrium* of fluids. But those that shall consider how when once an impetus is given to a fluid body, it is capable of mounting above its level, and checking others that have a contrary tendency to descend by their own gravity will no longer regard this as a material obstacle; but will rather conclude, that the great analogy there is between the rising and falling of the water upon the flux and reflux of the sea, and this of accumulating and extenuating the air, is a great argument for the truth of this hypothesis. For as the sea, over against the coast of Essex, rises and swells by the meeting of the two contrary tides of flood, whereof the one comes from the S. W. along the channel of England, and the other from the north, and on the contrary sinks below its level upon the retreat of the water both ways, in the tide of ebb; so it is very probable, that the air may ebb and flow after the same manner; but by reason of the diversity of causes whereby the air may be set in moving, the times of these fluxes and refluxes thereof are purely casual, and not reducible to any rule, as are the motions of the sea, depending wholly upon the regular course of the moon.

SECTION

SECTION VIII.

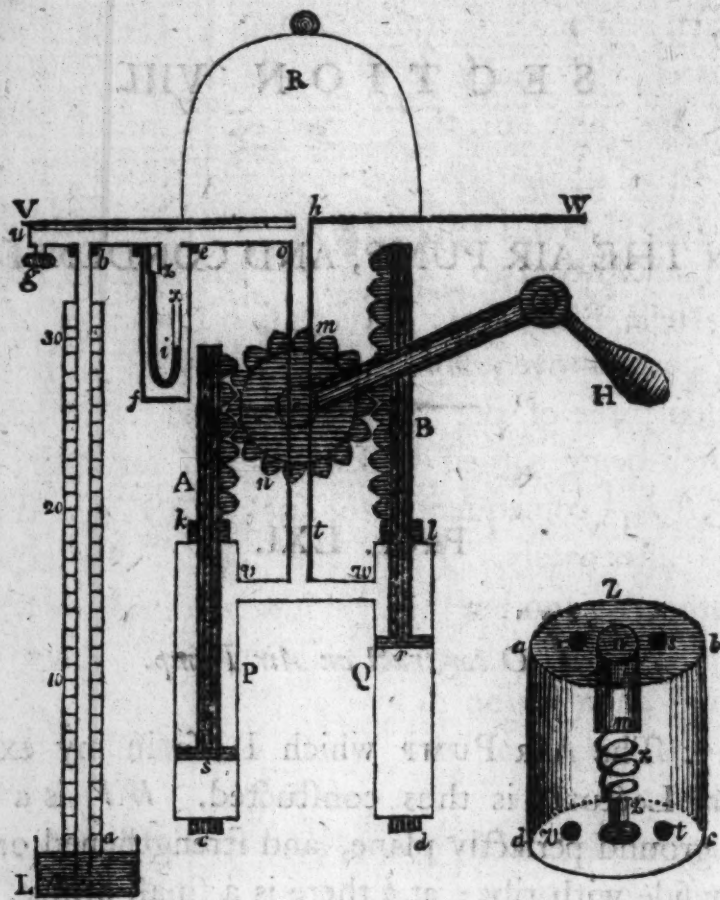
ON THE AIR PUMP, AND CONDENSER.

PROP. LXI.

To construct an Air Pump.

127. The AIR PUMP which I use in my experimental Lectures is thus constructed. *WV* is a brass plate ground perfectly plane, and strengthened on the under side with ribs; at *h* there is a small orifice, over which stands a glass vessel *R*, called a receiver, the edge of which is also ground truly plane, so that if a little grease be put upon the edge before it is placed on the receiver, it will be air-tight; in general however a piece of leather well prepared with grease is laid upon the plate for the receiver to stand upon; but you may make a more perfect exhaustion by the other method, on account of the air which the leather will give out; in this leather there is a hole made corresponding to *h* in the plate. From *h* a brass pipe *ht* descends,

descends, and turning each way at the bottom enters the two barrels, *P, Q*, at *v* and *w*. At the bottom of



each barrel there is a small hole against which there are two pieces *c, d* screwed on, containing the valves, each of which is represented by the figure *Z*, which is a solid piece *abcd* of brass, through the middle of which there is a cylindrical hole, partly filled with a solid brass cylinder, against the bottom *m* of which a spiral spring *x* acts, which rests below against a screw *z*, by means of which the spring may be rendered stronger or weaker; through the brass there are also

two

two holes, one from r to v , and the other from s to t ; and over the top ab there is tied a piece of oiled silk, having two holes corresponding to r and s ; and when this piece Z is screwed on to the bottom of the barrel, the end n of the cylinder nm is pressed against the hole in the barrel, by means of the spring x . The barrels are truly cylindrical, having each a sucker r, s , (without a valve) surrounded with leather, and fitted so close to the barrel as to be air-tight; these suckers are fixed to two brass rods A, B , having cogs above; mn is a small wheel with cogs acting on those of the rods, and moved by an handle H , which being turned backwards and forwards, the rods A, B and consequently the suckers s, r ascend and descend alternately. From the top of the pipe ht there proceeds another pipe ou , into an orifice of which there is fixed a glass tube ab , having its lower end immersed in a basin L of quicksilver; this tube is called the *gage*, at the back of which there is fixed a frame of wood, which is graduated from the mercury in the basin up to 31 inches. At g there is a screw, by unscrewing which you can admit the air into the pipe ou when it is exhausted. The rods A, B , pass each through a collar of leathers at k and l , which are air-tight. The supporters to the whole of this are here omitted, as they would have rendered the figure confused, and have been of no use for the understanding of the instrument. This being the construction, the exhaustion takes place in the following manner.

128. Turn the handle, and bring the sucker r down to the bottom of the barrel, then the sucker s will

emist

be carried just above the orifice *v*; and by turning the handle in the contrary direction, *s* will be depressed to the bottom, and *r* will rise just above the orifice *w*. Now upon the descent of *s*, it must manifestly force all the air in the barrel *P* before it, the sides being airtight; the air therefore will depress the cylinder *nm* (Fig. *Z*) and escape through the holes *rv*, *st*; after which the screw *x* will force *mn* up against the orifice at the bottom of the barrel, and prevent any air from returning into it. Then elevate *s* and depress *r*, and *r* in like manner will force out all the air before it. Now as *s* ascends, it leaves a vacuum between *s* and the bottom; but when *s* has gotten above *v*, the air will rush from the pipe *t*, which communicates with the receiver *R* and gage *ab*, into this vacuum, the consequence of which is, that the air in the receiver and gage becomes rarified by being expanded into a greater space; and as this must take place every time each sucker descends, or at each turn of the handle, there must be a continued exhaustion, and consequently a continued rarefaction of the air in the receiver and gage. But besides this gage, there is another included in a glass cylinder *ef* which has also a communication with the pipe *ou*; in this there is a bent glass tube *zix*, hermetically sealed at the upper end *z*, and filled with mercury to *i*, as represented by the shaded part. Then when the air is exhausted to a considerable degree, the pressure of the air upon the mercury at *i* will not be able to sustain the mercury in the other leg, and therefore it will descend, and the two surfaces will approach to the same level, and if you could make a perfect exhaustion, they would stand in the same

same horizontal line; the difference of the altitudes therefore (measured upon a scale which lies against them) shows how much there wants of a perfect vacuum. If to the height of the mercury in the other gage, you add the difference of the altitudes in this gage, it gives the altitude in the gage ab if you could make a perfect vacuum, or it gives the altitude at which the Barometer stands at that time. By this method you may try whether a Barometer be properly filled and graduated.

LEMMA.

129. Let a quantity a be diminished till it becomes successively $b, c, d, \&c.$ and let the decrements $a - b, b - c, c - d, \&c.$ be always in proportion to the quantities themselves $a, b, c, d, \&c.$ then will both these quantities and their decrements be in geometrical progression.

For by supposition, $a : a - b :: b : b - c :: c : c - d :: \&c.$ hence dividendo, $a : b :: b : c :: c : d :: \&c.$ Also alternando, $a : b :: a - b : b - c, b : c :: b - c : c - d, \&c.$ hence $a - b : b - c :: b - c : c - d :: \&c.$

PROP. LXII.

If b represent the capacity of one of the barrels, and r that of the receiver, together with the pipes and gages connected with it; then the quantity of air extracted after every turn : the quantity before that turn $:: b : 2b + r$; and the quantity left in : the quantity before $:: b + r : 2b + r.$

130. For conceive the sucker r to be down and s to be up, and the receiver, pipes, gages and barrels, which all now communicate, to be filled with air; then as the whole capacity of these is $2b+r$, the quantity of air may be represented by $2b+r$, from which, by the descent of s , the quantity b will be driven out; and this must evidently be the case at every turn. And as the quantity b is taken away from $2b+r$, there must remain the quantity $b+r$.

Cor. Hence the quantity taken away at every turn being always in the same ratio to the whole quantity before the turn, the air can never be all exhausted.

PROP. LXIII.

The density of the air in the receiver at first : the density after t turns :: $\overline{2b+r}^t : \overline{b+r}^t$.

131. For the density is (Art. 4.) as the quantity of air contained in the same space. Now the quantity before any turn : the quantity after :: $2b+r$: $b+r$ by Art. 130. and therefore the density at every turn is diminished in the same ratio; hence, by the composition of ratios, after t turns, the density is diminished in the ratio of $\overline{2b+r}^t : \overline{b+r}^t$.

Hence the density is diminished in geometrical progression.

PROP. XLIV.

When the density of the air is diminished in the ratio of

$$n : 1, \text{ the number of turns } t = \frac{\log. n}{\log. 2b+r. - \log. b+r.}$$

132. For

132. For (Art. 131) $n : 1 :: \overline{2b+r}^t : \overline{b+r}^t$, hence
 $n = \frac{\overline{2b+r}^t}{\overline{b+r}^t}$, consequently (Fluxions Art. 109) $\log. n =$
 $t \times \log. \frac{2b+r}{b+r} = t \times \log. \overline{2b+r} - \log. \overline{b+r}$; hence $t =$
 $\frac{\log. n}{\log. \overline{2b+r} - \log. \overline{b+r}}.$

PROP. LXV.

As the air is exhausted, the mercury will rise in the gage; and the defects of the mercury in the gage from the standard altitude, after each successive turn, form a geometric series, the ratio of whose terms is $2b+r : b+r$.

133. For as the density of the air within the gage, and consequently (Art. 122) it's compressing force on the mercury, is diminished at every turn, the compressing force of the air upon the mercury in the bason, which remains the same, must cause the mercury to rise in the gage. If all the air were exhausted, the mercury would rise as high as in the common Barometer, or to what is called the standard altitude. Now the compressing force of the quantity of air left in, prevents the mercury from rising to the standard altitude, and therefore it's compressing force must be equivalent to a column of mercury equal to the defect; therefore the defect, being as the compressing force, must be (Art. 122) in proportion to the density, which, at every turn, diminishes in the ratio of $2b+r : b+r$, by Art. 131.

PROP.

PROP. LXVI.

The ascents of the mercury in the gage, at each successive turn, form a geometric series, the ratio of whose terms is $2b+r : b+r$.

134. The defects of the mercury from the standard altitude diminish in the ratio of $2b+r : b+r$, and the differences of these defects are the successive ascents of the mercury; but, by the Lemma, if a set of quantities decrease in geometrical progression, their differences will also decrease in the same geometrical progression; hence the ascents of the mercury successively decrease in the ratio of $2b+r : b+r$.

135. The various properties of the air are very readily shown by the air pump; as in the following experiments:

Ex. 1. Air is necessary for the *production* of sound.

For if a bell be put under the receiver of an Air Pump, and the air be exhausted, the bell, when struck, cannot be heard; and if the air be gradually let in the sound will gradually increase.

Ex. 2. Air is necessary for the *propagation* of sound.

For if a receiver be put over a bell, and then another receiver over that, and the air be exhausted from between them, no sound is heard; the sound therefore is not propagated through the vacuum.

Ex. 3. Air is necessary for the existence of fire.

For

For if a candle be put under the receiver and the air be exhausted, it immediately goes out.

Ex. 4. Air is necessary for the existence of animal life.

For most animals put under the receiver die almost immediately upon exhausting the air, and probably all would, could we make a perfect vacuum.

Ex. 5. The pressure of the air is rendered visible, by taking away the air from one side of a body, whilst it continues on the other.

For if a bladder be tied over the top of a glass receiver, and the air be exhausted from within, at every exhaustion, the pressure of the air upon the bladder will continue to depress it, until it bursts with a very great explosion.

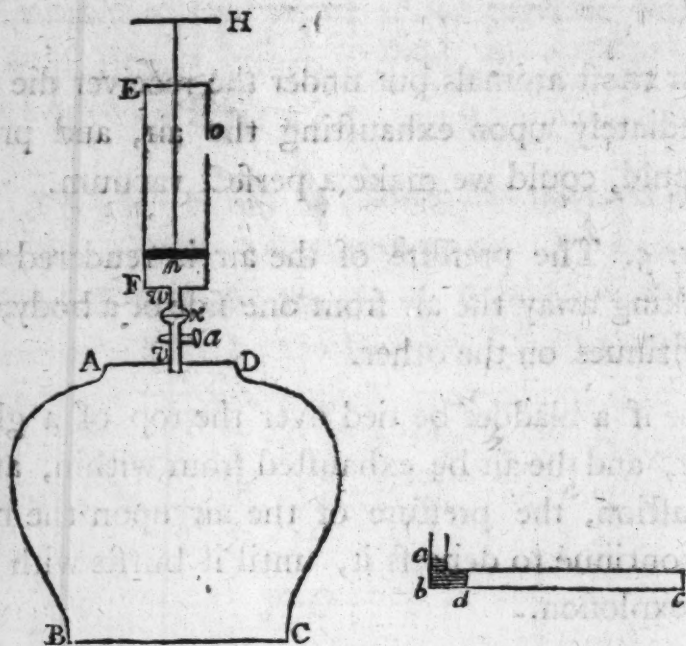
These are a few of the properties of the air which are shown by this instrument; but the experiments are too many to be all here enumerated.

PROP. LXVII.

To construct a Condenser.

136. A CONDENSER is thus constructed. *ABCD* is a strong vessel called a receiver, made either of glass or metal; if of glass, upon the top there is laid a brass plate with a stop cock *a*, having under it a prepared piece of leather to make it air-tight, and also a like plate at the bottom. Into the cock at *a* there is

is screwed a syringe EF , having a sucker n , which is moved by a handle at H ; at w there is a



valve which opens downwards, and at o there is an orifice. Now let the sucker be drawn up above the orifice o , and both the barrel of the syringe and the receiver to be filled with air in it's natural state. Then upon forcing down the sucker, the air opens the valve at w , and a barrel of common air is forced into the receiver. Upon raising again the sucker n a vacuum is left under it, the valve preventing the air from returning; and when the sucker gets above o , the air will immediately rush in and fill the barrel; thus upon every descent of the sucker you force into the receiver a barrel of common air, and consequently you condense the air in the receiver.

After

After the receiver is charged, the stop cock at a may be turned to prevent the return of the air, and the syringe may be taken off, and any other apparatus may be screwed on for experiments with the condensed air in the receiver.

PROP. LXVIII.

If b represent the capacity of the barrel of the syringe, and r that of the receiver, then after t descents of the sucker, the density of the air in the receiver will be to the density at first in the ratio of $r+tb : r$.

137. For the quantity of air at first may be represented by r , and after t descents of the sucker, a quantity represented by tb will be forced into the receiver, and therefore the whole quantity in it will be $r+tb$; hence, (Art. 4) the density after t descents : the density at first $:: r+tb : r$.

Cor. Hence the densities after any number of successive descents are in arithmetic progression.

138. If abc be a glass tube with the end at a open, and the other end hermetically sealed, and a small quantity of mercury put in so as to leave the air in dc in it's natural state; then if this be put into the receiver with the part bc horizontal, and the air be condensed, the condensed air pressing on the mercury will force it towards c , and the air in dc will continue of the same density as that in the receiver. Now as the density is inversely as the space occupied by the same quantity (Art. 36.) the density in dc , and consequently in the receiver, is inversely as dc ; when therefore dc is diminished until it be n times less than

than it was at first, the density will be increased n times. Hence, as the density, after any number of successive turns, increases in arithmetic progression, the reciprocal of the spaces will be in arithmetic progression, and therefore the spaces themselves will decrease in musical progression. This instrument is called a *gage*.

139. A bell in condensed air sounds louder than in air in its natural state. Fire Engines, Air Guns, Artificial Fountains, some kinds of Forcing Pumps, &c. act by condensed air.

SECTION

SECTION IX.

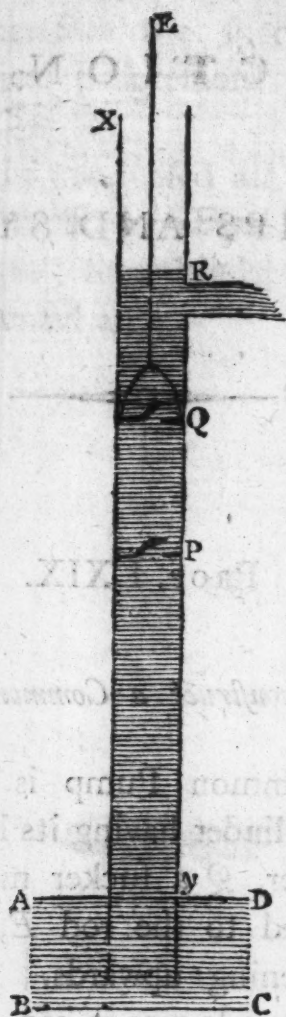
ON PUMPS AND SYPHONS.

PROP. LXIX.

To construct a Common Pump.

140. The common Pump is thus constructed. Xy is a hollow cylinder having its lower end in water, P is a fixed sucker, Q a sucker moveable by means of an handle fixed to the rod E , and each sucker has a valve opening upwards. Now let us suppose Q to descend as low as it can, and each valve to be shut, and that the pump has at present no water in it; then when Q ascends, the air between P and Q will follow it, and consequently it will become rarified, therefore the air under P being now denser than the air above, it will open the valve at P and rush into PQ , and the whole air within being thus rarified, it will not open the valve at Q , which is pressed down

down with air that is not rarified. The air therefore in the pump being rarified, the pressure of the air upon

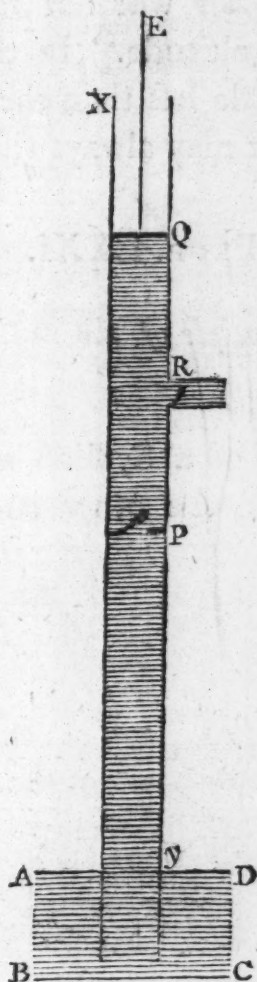


the surface of the water without the pump will force the water a little way up the pump. Now when Q descends, it will press down the air under it, and that air will shut the valve at P by pressing *upon* it, but it will open the valve at Q by pressing *under* it, and thus some of the air will escape. Then when Q ascends again, the

the pressure of the air upon it's valve will shut it, and the same operation will be repeated. Thus at each descent of Q the water will rise, till at length it comes up to Q , and then upon the descent of Q it will open it's valve and get above the sucker, and the sucker then being drawn up, it will carry the water up and throw it out of the spout R .

PROP. LXX.

To construct a forcing Pump.



141. Here the sucker Q has no valve, and the air
VOL. III. H between

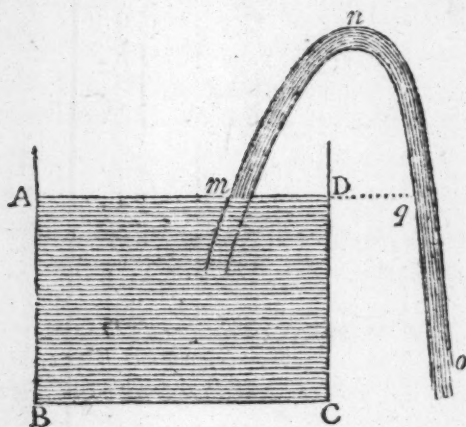
between P and Q is, by depressing the sucker Q , expelled through a valve opening outwards at R , instead of being expelled through Q , as in the other pump. Then when the water follows Q as Q ascends, upon it's descent it shuts the valve at P by pressing upon it, and opens that at R and forces out the water.

142. In this pump, Q must, at it's highest point, be within 32 feet of the water in the reservoir $ABCD$, because in the rarest state of the atmosphere, the pressure of the air will not raise the water in a vacuum above that altitude. In the other pump, P must be within a little less than the same distance, in order that the water may always rise above it.

PROP. LXXI.

To explain the principle of the motion of water through a Syphon.

143. If one end of a Syphon mno be put into a vessel of water, and the other end without be lower



than the surface of the water; then if the air be drawn out, the water will begin and continue to run
until

until the surface of the water in the vessel is on a level with the end o .

144. For when the air is drawn out of the Syphon, the water will rise in it to n by the pressure of the air upon the surface of the water in the vessel, and then it will descend to o by its gravity. Now the pressure of the air at o to force the water in the direction onm , is equal to the pressure of the air on the surface of the fluid in the vessel to force the water in the direction mno , at least extremely nearly so, on account of the very small difference of the altitudes of the air above m and o ; but the former pressure is opposed by the pressure of the column no , and the latter pressure is opposed by the pressure of the column mn ; the latter pressure of the air therefore being less opposed than the former pressure, the fluid must move in the direction of the latter pressure, or in the direction mno ; and the fluid will continue to run till the pressures of on , mn become equal, or till o and m are in the same horizontal line, for then their perpendicular heights being equal, their pressures will be equal by Art. 31.



SECTION X.

ON THE THERMOMETER, HYGROMETER, AND PYROMETER.

145. **A** THERMOMETER is an instrument constructed to measure different degrees of heat; it is a glass tube with a bulb at the bottom, having the bulb and part of the tube filled with a fluid; the tube is hermetically sealed at the top, and the part not occupied by the fluid is a vacuum. Against the tube there is a scale to measure the expansion of the fluid under different temperatures.

PROP. LXXII.

To find what Fluids are proper for Thermometers

146. Fluids expand by being heated, and contract again as they grow cold. Those fluids, therefore, which are not subject to be frozen, and whose expansion

expansion is sensible and in proportion to the heat applied, are proper for thermometers. Now the expansion of mercury, linseed oil, and spirits of wine, is, as to sense, proportional to the heat applied. This BROOK TAYLOR found by the following experiment. Having constructed a Thermometer with linseed oil, he put it into cold water, and then into water heated to any degree, and noticed the altitudes at which the fluid in the Thermometer stood in each case. He then put equal quantities of these waters together, which gave a mean heat; and by putting the Thermometer into this mixture, he found that it stood at a mean altitude between the two former altitudes. And this appeared to be true of whatever temperatures the two parts of water were. The mean temperature, therefore always agreeing with the mean altitude, the expansion must be in proportion to the heat. The same is found true of mercury and of spirits of wine.

PROP. LXXIII.

To fill a Thermometer.

147. The bore of the tube is so small that the fluid cannot be poured in; to get in the fluid, therefore, heat the bulb, by blowing the flame of a lamp against it with a blow-pipe, and you will expel the air from within; then dip the open end of the tube into the fluid, and it will rise up into the tube and bulb, by the pressure of the air upon the surface of the fluid into which you dip it, there being a vacuum, or nearly so, within the tube and bulb. If it do not fill the first time, repeat the operation till it does;

and if there be any air bubbles, tie a string to the end of the tube and whirl it about till the bubbles escape. Having thus filled the tube, hold it over the lamp till it boils, and in that state, let it be hermetically sealed, and upon the descent of the fluid, when it grows cold, the space above must be a vacuum.

PROP. LXXIV.

To graduate a Thermometer according to FAHRENHEIT'S scale.

148. Having filled the tube of the Thermometer, and fixed it against a frame upon which the graduations are to be made, put it into water just freezing, and against the surface put 32; then put it into boiling water, and against the fluid put 212; divide this interval into 180 equal parts, and also continue the same divisions down below 32 to the bulb. Then will 98 be blood heat, 76 summer heat, and 55 temperate. If the tube and scale be continued upwards to 600, it will give the heat of boiling mercury; and if it be continued downwards to 40 below 0, it will give the cold of freezing mercury. Or a thermometer may be graduated by comparing it with another, in this manner. Put them both into water, first of one temperature and then of another, and mark the ungraduated one in these two cases, according to the graduated one; then this interval may be subdivided, and the graduation continued both ways.

149. Hence a thermometer may be graduated for any other scale. In SIR I. NEWTON'S scale, freezing water is 0 and boiling water 34; and the other points
may

may be found by proportion from the other scale. For instance, to find blood heat on this scale, we may observe, that in Fahrenheit's thermometer, from freezing to boiling water is 180, and to blood heat 66; and in this scale, from freezing to boiling water is 34; hence $180 : 66 :: 34 : 12\frac{2}{3}$ the point of blood heat on this scale.

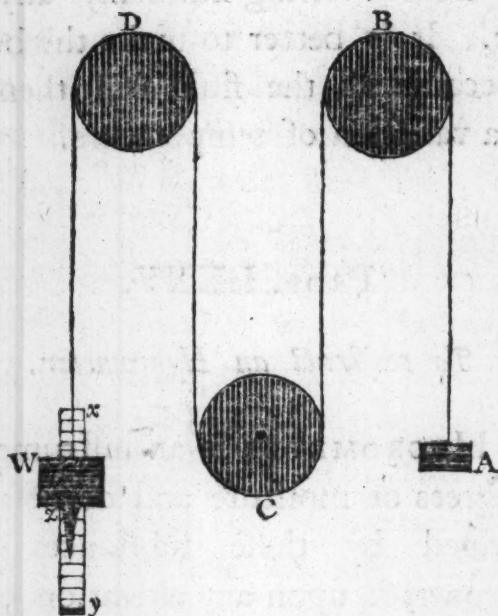
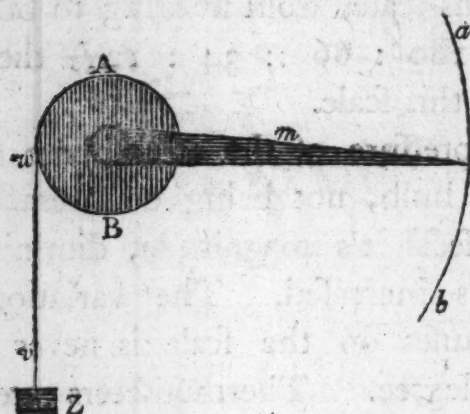
150. The pressure of the atmosphere against the outside of the bulb, not being counteracted by any air within, affects it's magnitude, diminishing it as the pressure is increased. The variation however which this causes on the scale is never above one tenth of a degree. Thermometers are generally made with spirits of wine or mercury, because linseed oil is found to adhere to the sides of the tube, which prevents it from showing suddenly any change of temperature. It is better to make the bulb flat than globular, because all the fluid will then be soonest affected by a variation of temperature.

PROP. LXXV.

To construct an Hygrometer.

151. An HYGROMETER is an instrument to determine the degrees of moisture and dryness of the air, and is formed by those substances which will expand or contract upon any alteration of the moisture. Wood expands by moisture and contracts by dryness; on the contrary, chord, catgut, &c. contract by moisture and expand by dryness. Various mechanical contrivances have been invented to render

sensible the smallest variations in the lengths of those substances. We will describe two of them.—Let AB be the section of a cylinder moveable about it's



axis, which is parallel to the horizon; at the end of which there is an index moveable against a graduated arc ab ; about this cylinder some catgut is wound, one end of which is fixed to the cylinder, and the other end

to

to something immoveable at *Z*. As the moisture of the air increases, the catgut contracts and turns the cylinder, and the motion of the index shows the increase of the moisture; and as the air decreases in moisture, the catgut will lengthen, and the weight of the index will carry the cylinder back, and the index will show the corresponding decrease of moisture. In the second figure, the catgut is fixed at *A* and goes over the pullies *B, C, D*, and at the other end a weight *W* is fixed, having an index *z* which moves against a graduated scale *xy*, that shows the increase and decrease of the length of the string, and consequently the state of the air in respect to it's moisture. Various other contrivances, upon the same principle, have been invented, but it would be foreign to the plan of this work to enter into a particular description of every instrument which has been constructed for this purpose.

152. Mr. DE LUC has made a great many experiments, in order to find out such substances as expand most nearly in proportion to the quantity of moisture imbibed. The result was, that whalebone and box, cut across the fibres, increase very nearly in proportion to the quantity of moisture, and more nearly so than any other substances which he tried. This he found by taking a quantity of shavings of each substance, and weighing them at the time when he measured the increase of the length of a slip of each, cut as above described, the increase of weight being always in proportion to the increase of length. In his construction of an Hygrometer he preferred the whalebone, first, on account of it's steadiness, in always coming to the same point at extreme moisture; secondly, on account of it's greater expansion, it increasing

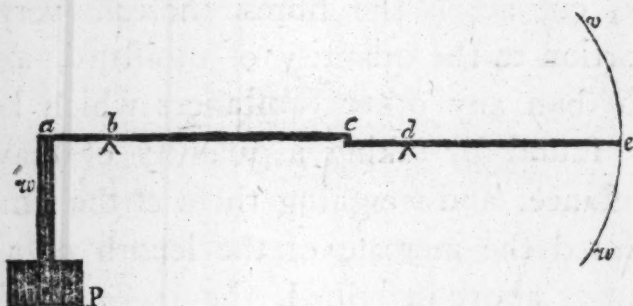
creasing in length above one eighth of itself from extreme dryness to extreme moisture; lastly, it is more easily made thin and narrow.

It is a little extraordinary, that when he took threads of some substances in the direction of the fibres, they first increased as the quantity of moisture increased, and afterwards upon a further increase of moisture they decreased in length. See the *Phil. Trans.* for 1791.

PROP. LXXVI.

To construct a Pyrometer.

153. A *Pyrometer* is an instrument invented to show the expansion and contraction of metals by heat and cold. Various machines have been constructed for this purpose; but as it would not be consistent with the plan of this work to enter into a particular description of each, we shall here only explain the general principle. Let *abc* be a lever



whose fulcum is *b*, acting upon another lever *cde*, whose fulcum is *d*; and let *w* be a metallic rod, one end of which rests against an immoveable obstacle *P*, and the other end against the lever *abc* at *a*. If a lamp

lamp be put under this rod, the heat will increase it's length, and put the levers in motion; now

$$\begin{aligned} \text{vel. of } a : \text{vel. of } c &:: ab : bc \\ \text{vel. of } c : \text{vel. of } e &:: cd : de \\ \therefore \text{vel. of } a : \text{vel. of } e &:: ab \times cd : bc \times de \end{aligned}$$

Hence if bc and de be very great in proportion to ab and cd , a small increase in the length of w will produce a considerable motion in the point e , which may be measured upon the graduated arc vw .

For example, if $ab : bc :: 1 : 25$, and $cd : de :: 1 : 40$, then $ab \times cd : bc \times de :: 1 \times 1 : 25 \times 40 :: 1 : 1000$; hence whilst the rod increases the 1000th part of an inch, the end e will describe 1 inch. On this principle the least increase of the length of the rod becomes visible. Instead of putting the lamp immediately under the rod w , this rod is laid upon another piece of metal, called the heater, and when the lamp has given this it's greatest degree of heat, the rod w is laid upon it.

154. In this manner Mr. MUSCHENBROEK made experiments to determine the proportion of the expansions of different metals, by applying a different number of lamps, and found the result as follows :

Lamps.	Iron.	Steel.	Copper.	Brass.	Tin.	Lead.
1	80	85	89	110	153	155
2	117	123	115	220	*	274
3	142	168	193	275	*	*
4	211	270	270	361	*	*
5	230	310	310	377	*	*

Tin

Tin melted with two lamps and lead with three. With this kind of pyrometer Mr. FERGUSON found the expansion of metals to be in the following proportion; iron and steel 3, copper $4\frac{1}{2}$, brass 5, tin 6, lead 7. An iron rod 3 feet long is about one 70th of an inch longer in summer than in winter.

155. If a metal be put into water and the water be heated, the metal expands as the heat of the water increases. By this method Mr. SMEATON determined the expansion of different metals; for by means of a mercurial thermometer immersed in the water he could always ascertain the degree of heat. He found that in equal intervals of time the expansions were in geometric progression. By this he was enabled to get the measure of the bar before it was applied to the instrument. This will be best understood by explaining an experiment. The time elapsed between applying the bar to the instrument and taking the first measure, was $\frac{1}{2}$ a minute; therefore the intervals between taking the succeeding measures were $\frac{1}{2}$ a minute also. The first measure was 208; the second 214,5; the third 216,5; the fourth 217,5. The differences of these are 6,5; 2; 1. Now these three numbers are nearly equal to 6, 3; 2, 25; 0, 8, which form a geometrical progression whose common ratio is 2,8. As therefore we may suppose the expansion from the instant the bar was applied to the time of taking the first measure followed the same law, we can find the expansion in the first $\frac{1}{2}$ minute (at the end of which the first measure was taken) by continuing back the progression, or multiplying 6,3 by 2,8, which gives 17,7 for the expansion the first $\frac{1}{2}$ minute; hence $208 - 17,7 = 190,3$ for the measure before

before the bar was applied. The following expansions are selected from Mr. SMEATON's table, showing how much a foot, in length, of each increases in decimals of an inch, by an increase of heat corresponding to 180 degrees of FAHRENHEIT's thermometer from freezing to boiling water. See Mr. SMEATON's account in the *Phil. Trans.* 1754.

White glass barometer tube	,01
Hard steel	,0147
Iron	,0151
Copper hammered	,0204
Cast brass	,0225
Grain tin	,0298
Lead	,0344
Zinc	,0353

156. Metals being thus subject to expansion by heat, a pendulum made with a single rod of metal will continually be subject to a variation in it's length from the variation of the temperature of the air. To correct this, Mr. HARRISON invented a pendulum, called a gridiron pendulum, composed of rods of steel and rods of brass, so connected together, that the brass expands upwards when the steel expands downwards; and by a proper adjustment of these rods, the distance from the point of suspension to the center of oscillation, may be rendered subject but to a very small variation. Mr. GRAHAM invented another method of preserving the length of the pendulum the same in different temperatures. He took a glass, or metallic tube, and put in some mercury; and the heat,

heat, which expands the glass or metal downwards, expands the mercury upwards; by the adjustment therefore of a proper quantity of mercury, he could make these effects in altering the length of the pendulum nearly destroy each other. He found the errors of a clock of this sort to be only about $\frac{1}{3}$ of the errors of the best clock of the common sort.



SECTION

SECTION XI.

ON WINDS, SOUND, VAPOURS, AND THE FORMATION OF SPRINGS.

PROP. LXXVII.

To explain the Causes of the various Winds.

157. **W**IND is a current of air, and it's direction is denominated from that point of the compass *from* which it blows. The principal, if not the only cause of wind is a partial rarefaction of the air by heat. When the air is heated, it becomes rarer, and therefore ascends; and the surrounding cold air rushing in to supply it's place, forms a current in some one direction. Winds may be divided into *constant*, or those which always blow in the same direction; *periodical*, or those which blow half a year in one direction, and half a year in the contrary direction; these are called *monsoons*; and *variable*, which are subject to no rules. The two former are also called *Trade* winds. We shall here give the principal phænomena of winds, from
Dr.

Dr. HALLEY's History thereof, in the *Philosophical Transactions*.

1st. In the *Atlantic* and *Pacific Ocean*, under the Equator there is a constant East wind.

2ly. To about 28° on each side of the Equator, the wind on the *north* side declines towards the north east; and the more so, the further you recede from the Equator; and on the *south* side, it declines in like manner towards the south east. The limits of these winds are greater in the *Pacific Ocean*, on the American, then on the African side, extending in the former case to about 32° , and in the latter to about 28° . And this is true likewise to the southward of the Equinoctial, for near the *Cape of Good Hope*, the limits of the trade winds are 3° or 4° nearer the line, than on the coast of *Brasil*.

3ly. Towards the *Caribbee* Islands, the aforesaid north-east wind becomes more and more easterly, so as sometimes to be east, and sometimes east-by-south, but mostly northwards of the east, a point or two.

4ly. On the coast of Africa, from the *Canaries* to about 10° . N. latitude, the wind sets in towards the north west; then it becomes south west, approaching more to the south as you approach the *Cape*. But away from the coasts, the winds are perpetually between the south and the east; on the African side they are more southerly; on the *Brasilian*, more easterly, so as to become almost due east. Upon the coast of Guinea, they are subject to frequent calms, and violent sudden gusts, called *Tornado's*, from all points of the compass.

5ly. In

gly. In the *Indian* ocean, the winds are partly *constant*, and partly *periodical*. Between *Madagascar* and *New Holland*, from 10° to 30° latitude, the wind blows south-east by east. During the months of *May*, *June*, *July*, *August*, *September*, *October*, the afore-said south-east winds extend to within 2° of the Equator; then for the other six months, the contrary winds set in, and blow from 3° to 10° S. latitude. From 3° S. latitude over the *Arabian* and *Indian* seas and *Bay of Bengal*, from *Sumatra* to the coast of *Africa*, there is another monsoon, blowing from *October* to *April* on the north-east point, and in the other half year from the opposite direction. Between *Madagascar* and *Africa*, a south-south-west wind blows from *April* to *October*, which, as you go more northerly, becomes more westerly, till it falls in with the west-south-west winds; but the Dr. could not obtain a satisfactory account, how the winds are in the other half year. To the eastward of *Sumatra* and *Malacca*, on the north side of the Equator along the coast of *Cambodia* and *China*, the monsoons blow and change at the same time as before-mentioned; but their directions are more northerly and southerly. These winds reach to the *Philippine Islands* and to *Japan*. Between the same Meridians, on the south side of the Equator, from *Sumatra* to *New Guinea*, the same monsoons are observed. The shifting of these winds is attended with great hurricanes.

158. The east wind about the Equator is thus explained. The sun moving from east to west, the point of greatest rarefaction of the air, by the heat of the sun, must move in the same direction; and

the point of greatest rarefaction following the sun, the air must continually rush in from the east and make a constant east wind.

159. The constant north-east wind on the north side of the Equator, and south-east wind on the south side, may be thus accounted for. The air towards the poles being denser than that at the Equator, will continually rush towards the Equator; but as the velocity of the different parts of the earth's surface, from it's rotation, increases as you approach the Equator, the air which is rushing from the north towards the Equator will not continue upon the same meridian, but it will be left behind; that is, in respect to the earth's surface, it will have a motion from the east, and these two motions combined produce a north-east wind on the north side of the Equator. And in like manner, there must be a south-east wind on the south side. The air which is thus continually moving from the Poles to the Equator, being rarefied when it comes there, ascends to the top of the atmosphere, and then returns back to the poles.

160. The cause of the *periodical* winds is supposed to be owing to the course of the sun northward and southward of the Equator. Dr. HALLEY explains them thus, "Seeing that so great Continents do interpose and break the continuity of the Ocean, regard must be had to the nature of the soil and the position of the high mountains, which I suppose the two principal causes of the several variations of the winds, from the former general rule: for if a country lying near the sun prove to be flat, sandy, low

low land, such as the *Desarts of Libya* are usually reported to be, the heat occasioned by the reflection of the sun's beams, and the retention thereof in the sand, is incredible to those that have not felt it; whereby the air being exceedingly rarefied, it is necessary that the cooler and more dense air should run thitherward to restore the equilibrium. This I take to be the cause, why near the coast of *Guinea* the wind always sets in upon the land, blowing westerly instead of easterly, there being sufficient reason to believe, that the inland parts of *Africa* are prodigiously hot, since the northern borders thereof were so intemperate, as to give the antients cause to conclude, that all beyond the *Tropic* was made uninhabitable by excess of heat. From the same cause it happens, that there are so constant calms in that part of the ocean, called the *rains*. For this tract being placed in the middle, between the westerly winds blowing on the coast of *Guinea*, and the easterly trade winds blowing to the westwards thereof, the tendency of the air here is indifferent to either, and so stands in equilibrio between both; and the weight of the incumbent atmosphere being diminished by the continual contrary winds blowing from hence, is the reason that the air here holds not the copious vapour it receives, but lets it fall into so frequent rains.

As the cool and dense air, by reason of its greater gravity, presses upon the hot and rarefied, 'tis demonstrative that this latter must ascend in a continual stream as fast as it is rarefied, and that being ascended it must disperse itself to preserve the equilibrium,

librium, that is, by a contrary current, the upper air must move from those parts where the greatest heat is: So by a kind of circulation, the N. E. trade wind below, will be attended with a S. W. above, and the S. E. with a N. W. wind above. And that this is more than a bare conjecture, the almost instantaneous change of the wind to the opposite point, which is frequently found in passing the limits of the trade winds, seems to assure us; but that which above all confirms this hypothesis is the phænomenon of the monsoons, by this means most easily solved, and without it hardly explicable. Supposing therefore such a circulation as above, 'tis to be considered that to the northward of the *Indian Ocean* there is every where land within the usual limits of the latitude of 30° , viz. *Arabia, Persia, India*, &c. which for the same reason as the mediterranean parts of *Africa* are subject to unsufferable heats when the sun is to the north, passing nearly vertical, but yet are temperate enough when the sun is removed towards the other tropic; because of a ridge of mountains at some distance within the land, laid to be frequently in winter covered with snow, over which the air, as it passes, must needs be much chilled. Hence it comes to pass, that the air coming, according to the general rule, out of the N. E. in the *Indian* seas, is sometimes hotter, sometimes colder than that which by this circulation is returned out of the S. W. and by consequence, sometimes the under current or wind is from N. E. sometimes from the S. W. as is clear from the times wherein these winds set in, viz. in *April*, when the sun begins to warm

warm those countries to the north, the S. W. monsoon begins, and blows during the heats till *October*, when the sun being retired, and all things growing cooler northward, and the heat increasing to the south, the N. E. winds enter and blow all the winter till *April* again.

And it is undoubtedly from the same principle that to the southward of the Equator, in part of the *Indian Ocean*, the N. W. wind succeeds the S. E. when the sun draws near the Tropic of *Capricorn*. But I must confess, that in this latter occurs a difficulty not well to be accounted for, which is, why this change of the monsoons should be any more in this Ocean, than in the same latitudes in the *Ethiopic*, where there is nothing more certain than a S. E. wind all the year.

'Tis likewise very hard to conceive, why the limits of the trade-wind should be fixt about the 30th deg. of latitude all round the globe; and that they should so seldom transgress or fall short of those bounds; as also that in the *Indian* sea, only the northern part should be subject to the changeable monsoons, and in the southern there be a constant S. E."

161. We may further add, that the causes mentioned in the last article, must here also operate. There may perhaps be some cases of these periodical winds, which we cannot see altogether a correct solution of; but if all the circumstances of situation, heat, cold, &c. were known, there is no reason to doubt but that they might be accounted for from the principles here delivered

162. We may further observe in respect to the direction in which winds blow, that if a current set off in any one direction, north-east for instance, and move in a great circle, it will not continue to move on that point of the compass, because a great circle will not meet all the meridians at the same angle, the meridians not being parallel. This circumstance must therefore enter into our consideration in estimating the direction of the wind. High mountains are also observed to turn the winds into a particular course. On the lake of *Geneva*, there are only two winds, that is, either up or down the valley. And the like is known to happen at other such places.

163. The *constant* and *periodical* winds blow only at sea; at land, the wind is always *variable*.

164. Besides the winds already mentioned, there are others called *Land* and *Sea Breezes*. The air over the land being hotter during the day, than the air over the sea, a current of air will set in from the sea to the land by day; but the air over the land being colder than that over the sea at night, the current at that time will be from the land to the sea. This is very remarkable in Islands situated between the tropics.

165. Mr. CLARE exemplifies this by the following experiment. In the middle of a vessel of water, place a water-plate of warm water, the water in the vessel representing the ocean, and the plate, the island rarefying the air over it. Then hold a lighted candle over the cold water, and blow it out, and the smoke will move towards the plate.

But

But if the plate be cold, and the ambient fluid be warm, the smoke will move in the contrary direction.

166. Dr. DERHAM, from repeated observations upon the motion of light, downy feathers, found that the greatest velocity of wind was not above 60 miles in an hour. But Mr. BRICE justly observes, that such experiments must be subject to great inaccuracy, as the feathers cannot proceed in a straight line; he therefore estimates the velocity by means of the shadow of a cloud over the earth; by which he found, that in a great storm the wind moved 63 miles in an hour; when it blows a fresh gale, at the rate of 21 miles an hour; and in a small breeze, at the rate of about 10 miles in an hour. But this method takes for granted, that the clouds move as fast as the wind. It is probable that the velocity is something more than what is here stated.

PROP. LXXVIII.

To explain the Nature of Sound.

167. *Sound* is a sensation excited by the vibrations of the air upon the tympanum or drum of the ear. That the air is the instrument by which sound is conveyed from the sonorous body is manifest from hence, that no sound can be produced if the body be in a vacuum, or if there be a vacuum between the body and the ear.

168. By percussion, the parts of a sonorous body, as a bell, a musical string, &c. are put into a state of

vibration, and as long as the vibrations are continued, corresponding vibrations are communicated to the air; and sound is heard, as long as the vibrations are strong enough to produce the sensation. All sonorous bodies are therefore elastic. The manner in which the vibrations are excited in the air is so clearly described by Mr. COTES, that I cannot do better than give the account in his own words. "The parts of the sonorous body, being put into a tremulous and vibrating motion, are by turns moved forwards and backwards. Now as they go forwards they must of necessity press upon the parts of the air to which they are contiguous, and force them also to move forwards in the same direction with themselves; and consequently those contiguous parts will at that time be condensed; then as the parts of the sonorous body return back again, the parts of the air which were just before condensed, will be permitted to return with them, and by returning they will again expand themselves. It is manifest therefore, that the contiguous parts of the air will go forwards and backwards by turns, and be subject to the like vibrating motion with the part of the sonorous body.

"And as the sonorous body produces a vibrating motion in the contiguous parts of the air, so will these parts thus agitated, in like manner produce a vibrating motion in the next parts, and those in the next, and so on continually. And as the first parts were condensed in their progress, and relaxed in their regress, so will the other parts, as often as they go forwards, be condensed, and as often as they go backwards, be relaxed. And therefore they will not
all

all go forwards together, and all go backwards together; for then their respective distances would always be the same, and consequently they could not be rarefied and condensed by turns; but meeting each other when they are condensed, and going from each other when they are rarefied, they must necessarily one part of them go forwards whilst the other goes backwards, by alternate changes from the first to the last.

“ Now the parts which go forwards, and by going forwards are condensed, constitute those pulses which strike upon our organs of hearing, and other obstacles they meet with; and therefore a succession of pulses will be propagated from the sonorous body. And because the vibrations of the sonorous body follow each other at equal intervals of time, the pulses which are excited by those several vibrations, will also succeed each other at the same equal intervals.”

169. As, when a fluid is put in motion, that motion is communicated in all directions, Sound must be propagated in all directions from a sonorous body as a center, in concentric superficies, or shells of air, called *Aerial Pulses*, or *Waves of Air*, analogous, as supposed by some, to the circular waves produced on the surface of water when a stone is thrown in. If the sound be impeded by a body which has a hole, the waves pass through, and diverge from it as a new center, and the sound is heard on all parts on the other side of the body.

170. The law by which the force of sound decreases as you recede from the sonorous body, is not easy to be determined by theory. It has been usually esti-

estimated, by dividing the surrounding air into shells of an equal thickness, and supposing these shells to act upon each other, as so many elastic bodies would; but it is probable that this is a supposition very far distant from the truth. The utmost distance at which a sound has been heard, is about 200 miles. This was observed in the war between England and Holland, in the year 1672. The unassisted human voice has been heard from Old to New Gibraltar, a distance of 10 or 12 miles; the watch-word, *All's well*, given at the latter, in a still night, having been heard at the former. In both these cases, the sound passed over the water; and it is found, that sound will always be conveyed much further along a smooth, than a rough surface.

171. The velocity of sound, produced by all bodies, is found by experiment to be 1142 feet in a second, subject to a small variation from the course of the wind. Dr. DERHAM determined this very accurately, by placing cannon at different distances, and firing them, and observing the interval between the flash and the report. And thus he also found that sound (or rather the pulses of air which excite it) moves uniformly; it being always found, that the interval was in proportion to the distance. Sir I. NEWTON determined the velocity of sound by Theory; with which if the reader wish to be acquainted, he may consult the *PRINCIPIA*, Lib. 2. Prop. 47. A very strong wind is found to alter the velocity of sound by about $\frac{1}{20}$ of the whole; to be added when the direction of the wind and sound coincide, and subtracted, when they oppose each other.

172. Sound

172. Sound is conveyed to the greatest distance by a trumpet, called a *speaking* or *stenterophonic* trumpet, the form of which is like that figure which would be generated by any part of the logarithmic curve revolving about it's axis, the mouth being applied to the smaller end. The theory by which this is attempted to be proved, is subject to the objection mentioned in Art. 170.

173. The *same* sound is always excited, when the air is put into the *same* state of vibration; that is, if a bell and musical string make the air vibrate the *same number* of times in a second, they excite the *same* tone. And as the same sonorous body performs all it's vibrations in the same time, whether greater or less, the same body will always give the same tone, whether the percussive stroke be greater or less. The slower the vibration, the deeper or graver is the tone. But we mean not here to enter into the investigation of the times of vibration of musical strings; a subject of considerable difficulty, and therefore not proper for an elementary treatise. If the reader wish for any information upon the subject, I refer him to Mr. PARKINSON'S *Hydrostatics*; or Dr. SMITH'S *Harmonics*.

174. The reflection of the vibrations of the air from any fixed object to the ear, will cause a sound distinct from that which is caused by the vibrations coming directly to the ear; and this is called an *Echo*. If the distance of the object which returns the echo be great, it will return several syllables. A single syllable will not be clearly returned unless the distance of the object be at least 120 feet; and so in proportion. Hence an echo returning ten syllables must
come

come from an object 1200 feet distant. More syllables however will be returned by night than by day, because the air being then colder, is denser, in which case, the return of the vibrations become slower, and consequently more syllables may be heard.

PROP. LXXIX.

To explain the ascent of Vapours, and the origin of Springs

175. *Vapours* are raised from the surface of the water; the principal cause of which is, probably, the heat of the sun, the evaporation being always greatest when the heat is the greatest. The difficulty of solving the phenomenon arises from hence, that we find a heavier fluid (water) suspended in a lighter fluid (air), contrary to our foregoing principles.

176. Dr. HALLEY supposed, that by the action of the sun upon the surface of the water, the aqueous particles become formed into hollow bubbles filled with warm, and rarefied air, so as to make the whole bulk specifically lighter than the air, in which case the particles will (Art. 45.) ascend. But there is a great difficulty is conceiving how this can be effected. And if bubbles could be *at first* thus formed, when they ascend, the air within would very soon be reduced to the same temperature of the air without, and they would immediately descend upon that effect taking place. Another opinion is, that the particles of water are separated by a repulsive force, which is increased in proportion as the heat is increased, and thus they are dispersed through the air; but the same argument

argument may be used against this hypothesis, as against the last, that is, that this effect could not continue in the cold part of the atmosphere where the clouds are suspended. The most probable supposition is, that evaporation is a chemical solution of air in water. We know that metals are dissolved in menstruums, and their particles diffused and suspended in the fluid, although their specific gravity be greater than that of the fluid. Heat promotes this solution; in the day time therefore the heat causes a more perfect solution than what can, *ceteris paribus*, take place in the night when the air is colder, when the heat is frequently not sufficient to keep the water in a state of solution, and it falls in fogs and dews. The vapours, thus raised by heat, ascend into the cold regions of the atmosphere, and not being there kept in a state of solution, they appear in the form of clouds; and when driven together by the agitation of the air, the particles run together into drops and fall down in rain. If they be frozen before they form themselves into drops, they descend in snow; but if the drops of rain themselves be frozen, they descend in hail. See HAMILTON on the *Ascent of Vapours*.

177. MARRIOTTE supposed *Springs* to be owing to rain water and melted snow, which penetrating the surfaces of hills, and running by the side of clay or rocks which it cannot penetrate, at last comes to some place where it breaks out. This would account for the phenomenon, provided the supply from these causes was sufficient; but D. SIDELEAU, and others, making an estimate of the quantity of rain and snow which falls in the space of a year, to see whether it would

would afford a quantity of water, equal to that which is annually discharged into the sea by the rivers (which are supplied by springs), found that it would *not*. But Dr. HALLEY discovered the cause of a sufficient supply; for he has proved by experiment, that the vapours which are raised, afford a much greater supply than is necessary; we will give the account in his own words.

178. "We took a pan of water (salted to the degree as is common sea-water, by the solution of about a fortieth part of salt) about four inches deep, and 7 inches $\frac{1}{2}$ diameter, in which we placed a thermometer, and by means of a pan of coals, we brought the water to the same degree of heat which is observed to be that of the air in our hottest summers; the thermometer nicely showing it. This done, we affixed the pan of water, with the thermometer in it, to one end of the beam of the scales, and exactly counterpoised it with weights in the other scale; and by the application or removal of the pan of coals, we found it very easy to maintain the water in the same degree of heat precisely. Doing thus, we found the weight of the water sensibly to decrease; and at the end of two hours we observed, that there wanted half an ounce troy, all but 7 grains, or 233 grains of water, which in that time had gone off in vapour; tho' one could hardly perceive it smok, and the water was not sensibly warm. This quantity in so short a time seemed very considerable, being little less than 6 ounces in 24 hours, from so small a surface as a circle of 8 inches diameter. To reduce this experiment to an exact *calculus*, and determine the thickness of the skin of water that had so evaporated, I assume
the

the experiment alledged by Dr. EDW. BERNARD to have been made in the *Oxford Society*, viz. that the cube foot *English* of water weighs exactly 76 pounds troy; this divided by 1728, the number of inches in a foot, will give $253\frac{1}{3}$ grains, or half ounce $13\frac{1}{3}$ grains for the weight of a cube inch of water; wherefore the weight of 233 grains is $\frac{233}{253}$, or 35 parts of 38 of a cube inch of water. Now the area of the circle, whose diameter is $7\frac{7}{8}$ inches, is 49 square inches; by which dividing the quantity of water evaporated, viz. $\frac{35}{38}$ of an inch, the quote $\frac{35}{1862}$ or $\frac{1}{53}$, shews that the thickness of the water evaporated was the 53^d part of an inch: But we will suppose it only the 60th part, for the facility of calculation. If therefore water as warm as the air in summer, exhales the thickness of a 60th part of an inch in two hours from it's whole surface, in 12 hours it will exhale $\frac{1}{10}$ of an inch; which quantity will be found abundantly sufficient to serve for all the rains, springs, and dews, and account for the *Caspian Sea's* being always at a stand, neither wasting nor overflowing; as likewise for the current said to set always in, at the *Straights of Gibraltar*, tho' those *Mediterranean Seas* receive so many, and so considerable rivers.

179. "To estimate the quantity of water arising in vapours out of the sea, I think I ought to consider it only for the time the sun is up, for that the dews return in the night as much, if not more vapours than are then emitted; and in summer the days being longer than
twelve

twelve hours, this excess is balanced by the weaker action of the sun, especially when rising before the water is warmed: So that if I allow $\frac{1}{10}$ of an inch of the surface of the sea to be raised *per diem* in vapours, it may not be an improbable conjecture.

“ Upon this supposition, every 10 square inches of the surface of the water yields in vapour *per diem* a cube inch of water; and each square foot, half a wine pint; every space of 4 feet square, a gallon; a mile square, 6914 tons; a square degree, suppose of 69 *English* miles, will evaporate 33 millions of tuns: And if the *Mediterranean* be estimated at 40 degrees long and 4 broad, allowances being made for the places where it is broader by those where it is narrower, (and I am sure I guess at the least) there will be 160 square degrees of sea; and consequently the whole *Mediterranean* must lose in vapour, in a summer's day, at least 5280 millions of tuns. And this quantity of vapour, though very great, is as little as can be concluded from the experiment produced: And yet there remains another cause, which cannot be reduced to the rule, I mean the winds, whereby the surface of the water is licked up, somewhat faster than it exhales by the heat of the sun, as it is well known to those that have considered those drying winds which blow sometimes.

“ The *Mediterranean* receives these considerable rivers; the *Iberus*, the *Rhone*, the *Tiber*, the *Po*, the *Danube*, the *Niester*, the *Borysthenes*, the *Tanais*, and the *Nile*, all the rest being of no great note, and their quantity of water inconsiderable. We will suppose

pose each of these nine rivers to bring down ten times as much water as the river *Thames*, not that any of them is so great in reality, but to comprehend with them all the small rivulets that fall into the sea, which otherwise I know not how to allow for.

“To caculate the water of the *Thames*, I assume that at *Kingston Bridge*, where the flood never reaches, and the water always runs down, the breadth of the channel is 100 yards, and it's depth 3, it being reduced to an equality, (in both which suppositions I am sure I take with the most.) Hence the profile of the water in this place is 300 square yards: This multiplied by 48 miles, (which I allow the water to run in 24 hours, at 2 miles in an hour) or 84480 yards, gives 25344000 cubic yards of water to be evacuated every day, that is 20300000 tons *per diem*; and I doubt not but in the excess of my measure of the channel of the river, I have made more than sufficient allowance for the waters of the *Brent*, the *Wandel*, the *Lea*, and *Darwent*, which are all worth notice, that fall into the *Thames* below *Kingston*.

“Now if each of the aforesaid nine rivers yield ten times as much water as the *Thames* doth, 'twill follow that each of them yields but 203 millions of tons *per diem*, and the whole nine but 1827 millions of tons in a day; which is but little more than $\frac{1}{3}$ of what is proved to be raised in vapours out of the *Mediterranean* in twelve hours time.”

180. Besides the *Constant Springs*, there are others which *ebb* and *flow* alternately, which may thus be accounted for. The water, before it breaks out, may meet with a large cavity on the side of the hill, and

the water, upon the overflowing of this reservoir, may find an aperture, and make it's escape; in case of dry weather, therefore, the supply of water may not be sufficient to keep it full, in which case, the spring will cease to flow, and continue dry, till a supply causes it overflow, and produce again the spring.

THE END OF VOL III.



ERRATA.

- PAGE 3, line 22, for *easy*, read *easy*.
 12, line 9, for *AB*, read *AD*.
 15, line 18, for $o \times z$, read $o \times o z$.
 19, line 11, for (*Art.* 17), read (*Art.* 18).
 26, line 14, for $W=5787$, read $W=,5787$.
 28, line 16, for (*Art.* 39), read (*Art.* 38).
 33, line 5, for (*Art.* 43), read (*Art.* 42).
 35, line 1, for *H*, read *P*.
 45, line 4, for *BCD*, read *fin. BCD*.
 48, line 6, for Cm^2 , read Pm^2 .
 75, line 5, for *a*, read *c*.
 76, last line, for $a : b :: b : c : c :: d : d :: \&c$:
 read $a : b :: b : c :: c : d :: d : \&c$.
 77, line 19, for (*PROP.* 57), read (*PROP.* 58).
 128, line 10, for *Pacific*, read *Atlantic*.

TABLE



